Indirect Network Effects
and Trade Liberalization

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Abstract

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary products available for use with an electronic hardware device. In this paper, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these purposes we construct a simple two-country model of trade with incompatible country-specific hardware technologies. It is shown that, given that both countries’ hardware

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devices remain in the trading equilibrium, both countries gain from trade liberalization. It is also shown that if only one country’s hardware remains in the integrated market, the other country may lose from trade liberalization.
1 Introduction

The proliferation of trade liberalization through both economic integration (e.g., the European Union) and preferential trade agreements (e.g., NAFTA) has spawned a vast literature on the implications of trade liberalization. As yet, however, little attention has been paid to the implications of trade liberalization in the presence of products with indirect (or virtual) network effects.

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary products available for an electronic hardware device. Examples of such devices include personal computers, video cassette recorders, and consumer electronics products. In systems that pair hardware with software, an indirect network effect arises because increases in the number of users of hardware increase the demand for compatible software and hence the supply of software varieties. Benefit to all consumers The consumers who purchase hardware/software systems thus constitute a virtual (or indirect) network.

Despite the fact that many industries characterized by indirect network

\footnote{A direct network effect can arise when increases in the number of consumers on the same network raise the consumption benefits for everyone on the network. The most common examples involve communications networks such as telephone and fax systems.}
effects are crucially related to trade liberalization, the literature on (indirect) network effects is almost exclusively focused on closed economies.\textsuperscript{2} Since the role of indirect network effects is amplified in the globalized world,\textsuperscript{3} it seems important to explore the impact of liberalization in the trade of products with indirect network effects.

As our primary contribution, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these purposes we construct a simple two-country model of trade with incompatible country-specific hardware technologies which is an extension of Church and Gandal’s (1992) closed-economy model. It is shown that, given that both countries’ hardware devices remain in the integrated market, both countries gain from trade liberalization. It is also shown that, if only one country’s

\textsuperscript{2}The seminal contributions on the role of a “hardware/software” system are Chou and Shy (1990, 1996), Church and Gandal (1992, 1996) and Desruelle et al. (1996). See Economides (1996), Shy (2001) and Gandal (2002) for surveys of the relevant closed-economy literature. For the open-economy context, Gandal and Shy (2001) analyze governments’ incentives to recognize foreign standards when there are network effects. The impact of trade liberalization, however, is downplayed in their analyses. See, also, Kikuchi (2007).

\textsuperscript{3}Gandal and Shy (2001, p. 364) note that, in 1992, it was estimated that seventy-two percent of all personal computers throughout the world were IBM-compatibles. That is, they ran the MS-DOS operating system and were compatible with applications software written for the MS-DOS operating system.
hardware remains in the integrated market, the other country may lose from trade liberalization.

The rest of the paper is organized as follows. Section 2 describes the basic model and derives autarky equilibrium. Section 3 describes the trading equilibrium. Section 4 considers gains and losses from trade liberalization. Section 5 contains concluding remarks.

2 The Model

Suppose that there are two countries in the world, Home and Foreign. In each country there are three types of goods: hardware, a large variety of software products, and the outside good. For hardware and we assume that there are country-specific hardware technologies: Home hardware and Foreign hardware. We also assume that the hardware technologies are incompatible: software written for one country’s hardware will not work with the other country’s. The characterization (i.e., location) of the two country-specific hardware technologies is exogenous: each is located at the end point of the unit line: let Home technology be at the left end point and Foreign technology at the right end point. We denote the marginal cost of hardware production in each country by \( c \), which implies there are no sources for comparative advantage. We further assume that the hardware technologies
are non-proprietary and that they will be offered at marginal cost. In this section, we consider the Home autarky situation where only Home hardware is available.

Consumer preferences over the combination of hardware and software are modelled as a Dixit-Stiglitz (1977) CES utility function.\(^4\) We assume that the distribution of the tastes of Home consumers is decreasing along a line of unit length \( t \in [0, 1] \). We also assume that the density of type \( t \) consumers in Home is \( 1 - t \): the total number of Home consumers is \( 1/2 \). Consumer densities are mirror images of each other: in Foreign, the density of type \( t \) consumers is \( t \).

The preferences of a consumer of type \( t \) for a system are:

\[
U(t) = \left[ \sum_{i=1}^{n} (x_i)^{\theta} \right]^{(1/\theta)} + \phi - kt, \quad 1/2 < \theta < 1, \tag{1}
\]

where \( n \) is the number of software products written for the Home hardware, \( x_i \) is the level of consumption of software product \( i \), \( \sigma \equiv 1/(1 - \theta) > 2 \) is the elasticity of substitution between every pair of software products, and we assume that \( \phi > k \). This specification of preferences incorporates the assumption that variety is important.

The representative consumer who purchases the hardware will maximize

\(^4\)See, also, Chou and Shy (1990) and Church and Gandal (1992).
subject to the following budget constraint:

\[ \sum_{i} p_i x_i = e - c, \]  

where \( p_i \) is the price of Home software variety \( i \), \( e \) is the total expenditure allocated to hardware and software, and \( c \) is the price (i.e., cost) of a unit of Home hardware.

The solution to this problem consists of the following demand functions:

\[ x_i = (e - c) p^{\sigma-1}_i / p_i^\sigma, \]  

where

\[ P = [\sum_{j=1}^{n} (p_j)^{1-\sigma}]^{1/(1-\sigma)}. \]

The indirect utility of a type-\( t \) consumer who purchases a Home system is

\[ V(t) = n^{1/(\sigma-1)}(e - c)/p + \phi - kt. \]  

The indirect utility function is concave in \( n \): the marginal benefit of another software variety is decreasing.

Now, turn to the cost structure of software production. The technology for the production of software is characterized by increasing returns to scale, since software creation typically involves fixed costs. We denote the constant marginal cost of software production for every product by \( b \), and the software development cost by \( f \).
We assume that software firms are monopolistic competitors, and thus, each product is priced at a markup over marginal cost \( b \):

\[
p = b\sigma/(\sigma - 1).
\] (6)

Rational expectations provide software for providers choose to provide for the Home hardware. Software product developed for that systems. Then the profit of a Home software firm is\(^5\)

\[
\pi = (p - b)(x/2) - f,
\] (7)

where \( x = (e - c)/np \). In the autarky situation in which only Home hardware exists, all Home software firms choose to provide software that is compatible with Home hardware and the number of Home software firms is determined via free entry as follows:

\[
n^A = (e - c)/2f\sigma,
\] (8)

where \( A \) refers to the autarky value.

The indirect utility of a consumer located at \( t \) is

\[
V^A(t) = (n^A)^{1/(\sigma - 1)}(e - c)(\sigma - 1)/b\sigma + \phi - kt.
\]

Consumer welfare in the autarky situation is

\[
W^A = \int_0^1 [((n^A)^{1/(\sigma - 1)}(e - c)(\sigma - 1)/b\sigma + \phi - kt)(1 - t)dt
\]

\[
= (1/2)[(n^A)^{1/(\sigma - 1)}(e - c)(\sigma - 1)/b\sigma + \phi] - k/6. \tag{9}
\]

\(^5\)Note that the total size of the Home consumer is 1/2.
3 Trading Equilibrium

The commencement of trade implies two basic changes in the market: (a) both Home and Foreign hardware devices are available to all consumers, and (b) the distribution of consumers’ tastes is uniform along the line and the total number of consumers becomes 1.

The timing of the game is as follows: In the first stage software firms enter the industry. There is free entry into the software industry and software firms have rational expectations. Although there may be more than one equilibrium software configuration, we show that the free-entry number of software firms, \(N = n + n^*\), is unique, where \(n\) and \(n^*\) are the number of firms providing software for Home and Foreign hardware, respectively. In the second stage, software firms simultaneously choose which platform to provide software for. In the final stage, each consumer purchases either a Home or a Foreign hardware system and some of the compatible software. We solve this problem backward.

3.1 Final Stage

Since we assume the marginal costs (prices) of hardware and software are equal for both systems, consumers determine which hardware to purchase.

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\[\text{This is taken from Church and Gandal’s (1992) closed-economy model.}\]
considering only their tastes and the amount of software available for each system. From (5), a consumer located at \( t \) purchases Home hardware if the following inequality holds:

\[
n^{1/(\sigma-1)}(e-c)/p + \phi - kt > (N-n)^{1/(\sigma-1)}(e-c)/p + \phi - k(1-t),
\]

where use has been made of the equation \( n + n^* = N \). Therefore, the location of the marginal consumer who purchase Home hardware is given by a function of \( n \), that is,

\[
t(n) = [n^{1/(\sigma-1)} - (N-n)^{1/(\sigma-1)}](e-c)(\sigma - 1)/(2kb\sigma + 1/2).
\]

And the first derivative of \( t(n) \) is positive:

\[
t'(n) = \frac{dt(n)}{dn} = \left[ \frac{n^{(2-\sigma)/(\sigma-1)} + (N-n)^{(2-\sigma)/(\sigma-1)}(e-c)}{2kb\sigma} \right] > 0.
\]

This means that the share of Home hardware is increasing in the amount of software for it. It can also be shown that

\[
t(0) \geq 0 \quad \text{and} \quad t(N) \leq 1 \iff N^{1/(\sigma-1)} \leq \frac{kb\sigma}{[(e-c)(\sigma - 1)]}
\]

and

\[
t'(N/2) \geq 1/N \iff N^{1/(\sigma-1)} \geq \frac{2^{1/(\sigma-1)}kb\sigma / 2(e-c)}{2(e-c)}.
\]
Based on the above, we can draw the function $t(n)$ as shown in Figure 1, where curves A, B, and C correspond to the graph of $t(n)$ under each of the following three cases: in case A, $N^{1/(\sigma-1)} \leq k\sigma / [(e - c)(\sigma - 1)]$; in case B, $k\sigma / [(e - c)(\sigma - 1)] < N^{1/(\sigma-1)} < 2^{1/(\sigma-1)} k\sigma / 2(e - c)$; and in case C, $N^{1/(\sigma-1)} \geq 2^{1/(\sigma-1)} k\sigma / 2(e - c)$.\(^7\)

**Figure 1**

Note that in cases B and C, $t(n)$ can reach 0 or 1, even if there are still two types of software. Since the market is of unit length, that is, $0 \leq t \leq 1$, there exists a critical number of software firms for each type of hardware such that if the number of software firms for one technology exceeds the critical number, then all consumers purchase the dominant hardware. On the other hand, in case A, there are two types of consumers unless one hardware is standardized; no software for the other hardware exists.\(^9\)

\(^7\)The second derivative of $t(n)$ is negative (positive) if $n$ is smaller (greater) than $N/2$, since

$$
\frac{d^2 t(n)}{dn^2} = -\frac{[n^{(3-2\sigma)/(\sigma-1)} - (N - n)^{(3-2\sigma)/(\sigma-1)}](\sigma - 2)(e - c)}{2k\sigma(\sigma - 1)},
$$

where $\sigma > 2$ from the assumption $\theta > 1/2$.

\(^8\)The importance of discrimination between case B and C will appear in the following.

\(^9\)Since we assume that hardware only facilitates the consumption of software and provides no stand-alone benefits, in case A, the marginal consumer, $t$, changes discontinuously to 0 or 1 when $n$ is equal to 0 or $N$. 

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3.2 Second Stage

In the second stage, software firms simultaneously select the network for which to supply software are. Given the marginal consumer, \( t \), and the number of competing software firms (\( n \) or \( n^* \)), the profit of a software firm writing software for Home hardware is

\[
\pi(t, n) = t(p - b)x - f = t(e - c)/n\sigma - f, \tag{15}
\]

and that for Foreign hardware is

\[
\pi^*(t, n^*) = (1 - t)(p - b)x^* - f = (1 - t)(e - c)/n^*\sigma - f, \tag{16}
\]

where \( x^* = (e - c)/n^*p \). From these equations, it is easily derived that

\[
\pi(t, n) > \pi^*(t, n^*) \iff t > \frac{n}{N}. \tag{17}
\]

Based on the latter inequality, each firm considers whether \( t(n) \) is greater than \( n/N \) or not, and then chooses the network to supply.

3.2.1 First Stage

At any equilibrium where two networks coexist, \( \pi(t, n) = \pi^*(t, n^*) \) must be satisfied. Therefore, \( t = n/N \) holds at the equilibrium and

\[
\pi = \pi^* = (e - c)/N\sigma - f. \tag{18}
\]
On the other hand, if all software firms provide software for one network at equilibrium, then \((t, n) = (1, N)\) or \((t, n^*) = (0, N)\) hold and

\[
\pi = (e - c)/N\sigma - f \quad \text{or} \quad \pi^* = (e - c)/N\sigma - f. \tag{19}
\]

Thus, the profit of each firm is independent of equilibrium software configurations, and the free-entry number of firms, \(N\), is uniquely given by \(N = (e - c)/f\sigma\) from the zero-profit condition.

Based on the foregoing argument, we can conclude that \(\pi = \pi^* = 0\) holds for any pair \((t, n)\) on the dotted line in Figure 1, \(\pi = 0\) at \((1, N)\), and \(\pi^* = 0\) at \((0, 0)\), while \(\pi\) (\(\pi^*\)) is positive (negative) at any pair above the line and \textit{vice versa}.

### 3.3 Nash Equilibrium Configurations

Based on the foregoing argument, we obtain the Nash equilibrium configurations as follows: In order for a configuration to be a Nash equilibrium, it must be impossible for a software firm to switch networks and increase its profit.

In case A, the graph of \(t(n)\) is drawn as curve A in Figure 1. So, there are three equilibrium candidates; \((n = n^* = N/2)\), \((n = N, n^* = 0)\), and
\((n = 0, n^* = N)\). Since

\[
\begin{align*}
t(n) \begin{cases} 
> n/N & \text{if } n < N/2, \\
< n/N & \text{if } n > N/2,
\end{cases}
\end{align*}
\]

we can conclude that only symmetric equilibrium \((n = n^* = N/2)\) is stable in the sense of a Nash equilibrium.

On the other hand, in case \(C\), the graph is drawn as curve \(C\) and

\[
\begin{align*}
t(n) \begin{cases} 
< n/N & \text{if } n < N/2, \\
> n/N & \text{if } n > N/2.
\end{cases}
\end{align*}
\]

Therefore, only two equilibria, \((n = N, n^* = 0)\) and \((n = 0, n^* = N)\), are stable.\(^{10}\)

Finally, in case \(B\), the graph of \(t(n)\) is drawn as curve \(B\) and it is apparent from the discussion above that all three of the equilibria, \((n = n^* = N/2)\), \((n = N, n^* = 0)\), and \((n = 0, n^* = N)\), are stable. So, we have the following lemma:

**Lemma:** Depending on the parameter values, the following three cases emerge:

**Case A:** If \(N^{1/(\sigma-1)} \leq k\sigma/[\sigma((e-c)(\sigma-1))]\), a unique symmetric equilibrium exists, \((n = n^* = N/2)\).

\(^{10}\)In the interval of \(n\) where \(t(n)\) is greater than 1 (smaller than 0), the actual marginal consumer, \(t\), is equal to 1 (0) and is still above (below) the line \(t = n/N\).
Case B: If \( kb/[(e - c)(\sigma - 1)] < N^{1/(\sigma - 1)} < 2^{1/(\sigma - 1)}kb\sigma/2(e - c) \), three equilibria, \((n = n^* = N/2), (n = N, n^* = 0), \text{ and } (n = 0, n^* = N)\), exist.

Case C: If \( N^{1/(\sigma - 1)} \geq 2^{1/(\sigma - 1)}kb\sigma/2(e - c) \), only two equilibria, \((n = N, n^* = 0)\) and \((n = 0, n^* = N)\), exist.

4 Gains/Losses from Trade

We now consider the welfare aspects of trade liberalization. If both countries’ hardware devices (and hence software written for each type of hardware) remain in the equilibrium (i.e., cases A and B), the indirect utility of Home consumers who purchase Home hardware (i.e., \( 0 \leq t \leq (1/2) \)) also remains unchanged.\(^{11}\) Furthermore, consumers who switch from Home hardware to Foreign hardware \((1/2 < t \leq 1)\) obtain decreased disutility, since a type-\( t \) consumer who switches obtains \( tk - (1 - t)k \) and the total gains from this switch are \( \int_{1/2}^{1}[kt - k(1 - t)](1 - t)dt = k/24.\(^{12}\) The same thing occurs in Foreign: consumers who switch from Foreign hardware to Home hardware obtain decreased disutilities.

\(^{11}\)Note that the amount of software for the Home hardware device remains unchanged: \( n = n^A = N/2.\)

\(^{12}\)Since \( n = n^* = n^A \), the availability of software remains unchanged by switching hardware devices.
**Proposition 1:** If both countries’ hardware devices remain in the equilibrium, both countries gain from trade.

Note that these gains correspond to those obtained from the “ideal-variety” approach to trade gains (e.g., Helpman and Krugman, 1985). By opening trade, consumers in each country can obtain goods (i.e., hardware) which are close to their “ideal” type, which constitutes mutual trade gains.

Now turn to the case of standardization (i.e., the acceptance of a single type of hardware). In cases B and C, there is some possibility that Home hardware (and complementary software) will vanish in equilibrium.\(^\text{13}\) In such a case, the Home welfare level becomes

\[
W^T = \int_0^1 [(N)^{1/(\alpha - 1)}(e - c)(\sigma - 1)/b\sigma + \phi - k(1 - t)](1 - t)dt = (1/2)[(N)^{1/(\alpha - 1)}(e - c)(\sigma - 1)/b\sigma + \phi] - k/3. \tag{22}
\]

While there are gains from increased diversity of software provision, there are losses from switching to the Foreign network. Comparing (9) and (22), we can obtain the critical condition for *losses from trade*:

\[
(k/6) > (1/2)[(1/2)^{1/(\alpha - 1)} - 1]N^{1/(\alpha - 1)}(e - c)(\sigma - 1)/b\sigma. \tag{23}
\]

\(^{13}\)If Home hardware dominates the market, Home consumers clear gain from trade liberalization due to increased (doubled) software diversification.
Note that the LHS indicates increased disutility (i.e., costs) from hardware switching, while the RHS indicates increased utility (i.e., gains) from software diversification.

**Proposition 2:** If one country’s hardware dominates the integrated market and condition (23) holds, the other country loses from trade liberalization.

These results are summarized in Figure 2.\(^{14}\) The possibility of loss from trade is shown as a shaded area. This implies that trade liberalization leads consumers to “switch” to a Foreign-dominated brand, thereby increasing the aggregate disutility. It is important to note that the losses from trade liberalization occur only if there are multiple equilibria.\(^{15}\) Note also that this does not mean that all Home consumers lose from trade. Home consumers who are located near 1 unambiguously gain from trade.

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\(^{14}\)Inequality (23) yields \(N^1/(\sigma-1) < \eta \equiv [1/3(1-2^1/(1-\sigma))]k\sigma/[(e-c)(\sigma-1)].\) We can show that if \(\sigma > \ln 3/\ln(3/2)\) then \(\eta\) is greater than \(k\sigma/[(e-c)(\sigma-1)],\) while it is always smaller than \(2^{1/(\sigma-1)}k\sigma/2(e-c).\)

\(^{15}\)This finding is consistent with Farrell and Saloner’s (1986) results on *excess standardization* in their closed-economy model.
5 Conclusions

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary products available for a hardware device. In this paper, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these purposes we construct a simple two-country model of trade with incompatible country-specific hardware technologies. It is shown that, given that both countries’ hardware devices remain in the trading equilibrium, both countries gain from trade liberalization (Proposition 1). It is also shown that, if only one country’s hardware remains in the integrated market, the other country may lose from trade liberalization (Proposition 2).

The present analysis must be regarded as tentative. Hopefully it provides a useful paradigm for considering how indirect network effects (or hardware/software systems) affect both the structure of international trade and the gains or losses from trade liberalization.

References

259–270.


Figure 1
\[
\frac{k_b \sigma}{(e-c)(\sigma-1)} \quad \eta \quad \frac{2^{1/(\sigma-1)}k_b \sigma}{2(e-c)}
\]

Figure 2