PACKAGING LICENSES IN PATENT POOLS*

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Patent pools are organizations where patent holders concentrate their own patents and offer licenses to each other and third parties. Most of the literature on patent pools has analyzed the single package license, which includes all the patents in the pool (Lerner and Tirole 2004; Shapiro 2001). However, to date there has been no study of multiple package licenses, which are packaged within subsets of all the patents in the pool. This paper develops a model that can analyze the multiple package licenses offered by a patent pool and discusses multiple package licenses from an antitrust perspective.

Keyword: patent pools, multiple package licenses, antitrust laws.

1. Introduction

This paper investigates the anticompetitive effects of a patent pool that offers a package license to users. Our analysis is characterized by two types of package licenses: the single package license and the multiple package license. The single package license is inclusive of all patents in the patent pool. If the single package license is offered, users can use all of the patents in the patent pool to commercialize new innovations. The multiple package license includes a subset of the patents in the patent pool. If the multiple package license is offered, users can select a license that only includes the patents they require from the patent pool.

A patent pool refers to organizations where patent holders concentrate their own patents for commercializing new innovations or for setting standards, and offer a package license that is inclusive of many of the patents in the pool. A patent pool plays an important role in solving the “tragedy of the anticommons,” which is discussed in Heller and Eisenberg (1998). The well-known “tragedy of the commons” is the situation wherein a resource can be overused when it is not protected by property rights. “Tragedy of the anticommons,” as Heller and Eisenberg indicate, refers to a situation wherein “excessive” property rights render the resource underused when there are multiple property rights holders. In the case of patents, excessive property rights can have the perverse effect of stifling or discouraging innovation. A patent pool is expected to be a useful means to solve this “tragedy of the anticommons”, which particularly arises in advanced technology fields. A patent pool enables firms to reduce the cost of seeking technologies and negotiating by simplifying the license agreement. Furthermore, a patent pool can avoid patent litigations and can help establish standardization committees such

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1) Examples of comprehensive surveys on patent pools are Shapiro (2001) and Gilbert (2004).
as MPEG-LA, DVD 6C Licensing Agency, and 3G.\(^2\)

Unfortunately, the competition authorities in many countries have a deep-rooted suspicion of patent pools, which involve cooperative activity between patent holders. There is the possibility that a patent pool can exercise monopoly power as a cartel.\(^3\) Historically, patent pools have been abused since the early 1900s.\(^4\) Priest (1977) indicates that it is possible that a patent pool is a means to disguise a cartel, formed by using a cross-license between the members in the pool. Thus, many discussions have been held between economists, legal scholars, and antitrust enforcement leagues as to whether patent pools benefit both intellectual property owners and consumers. Our concern is to determine whether patent pools are competitive or anticompetitive.

The US competition authority focuses primarily on the technical relationships between the patents included in the pool.\(^5\) Its viewpoint is that a pool of technically substitutable patents is more suspicious than a pool of technically complementary patents. In addition to this view of the US antitrust enforcement agency, Shapiro (2001) and Lerner and Tirole (2004) focus on the technical relationships between the patents included in a pool, and investigated whether patent pools have an anticompetitive effect. They conclude that a patent pool is pro-competitive when the patents are technical complements, whereas a patent pool always operates as a cartel when the patents are technical substitutes. These results are consistent with the current US and European policies (see Lerner and Tirole 2007).

Most of the literature on patent pools has sought to determine the social implications of a pool in the situation where all firms have joined the pool (Lerner and Tirole 2004). Recently, some works have focused on the firm’s incentive problem in relation to participation in the pool (Aoki and Nagaoka 2004; Brenner 2009; Langinier 2006; Lerner and Tirole 2007). Our paper considers both a patent pool that offers only a single package license and a patent pool that offers multiple package licenses. In the literature, there are no theoretical discussions of patent pools that offer both single package and multiple package licenses. The multiple package licenses are packaged within the subsets of all the patents in the pool. In practice, it is observed that about 12% of the pools surveyed by Lerner et al. (2003) offer multiple package licenses (that is, about 88% of the pools offer single package licenses). For example, MPEG-LA, which is the patent pool administrator for MPEG compression technological standards, offers multiple package licenses. One enormous advantage of multiple package licenses is that they give users various choices of patents, because a single package license that is inclusive of all the patents in the pool could be a tie-in sale. The recent guidelines of the European Commission encourage patent pools to offer multiple package licenses as a useful way to

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2) Standardization committees are organizations that set international technological standards. Technological standards aim at the wide adoption of technologies in the marketplace. However, this wide adoption of technologies may bring about patent conflicts between patent holders or firms. Standardization committees establish patent pools in order to avoid these conflicts. MPEG-LA, DVD 6C Licensing Agency, and 3G are examples of standardization committees.


5) The US Department of Justice focused on the technical relationship between pools in three business review letters regarding an MPEG patent pool and two DVD patent pools. See Shapiro (2001) for details.
provide users with a broader choice. In this paper, we investigate patent holders’ incentives to form a pool, the patent pool’s licensing behavior, and the anticompetitive effects of the pool. In particular, we focus on a patent pool that offers multiple package licenses. What characteristics does a patent pool that offers multiple package licenses have? Is a patent pool that offers multiple package licenses efficient? We find that the technical relationship between the patented technologies in a pool plays a critical role in answering the above questions. It is concluded that a patent pool is pro-competitive when the patents are technical complements, whereas a patent pool is anti-competitive when the patents are technical substitutes.

The paper is organized as follows. Section 2 defines the users’ gross surplus for using patents and the technical relationship between the patented technologies in our model: complements and substitutes. Section 3 characterizes licensing fees in the case where the patent pool licenses the patented technologies to users monopolistically. Section 4 characterizes licensing fees in the case where patent holders license the patented technologies to users individually. Section 5 analyzes the anticompetitive effects of a patent pool, using the outcomes of Sections 3 and 4, and then investigates patent holders incentive to form a patent pool. Finally, we discuss the results derived from the analysis conducted in the paper.

2. Basic Model

The basic set up of our model is the same as Shapiro (2001), and Lerner and Tirole (2004). We suppose that there are two different but symmetric patents, A and B. Each patent is owned by a patent holder A and B. We distinguish between patent holders and users. Patent holders do not have the ability to commercialize the patented technology on their own. The patent holder obtains a licensing fee from users, and users obtain a surplus by using the patents of the patent holders.

Users make their products by using the patents, which are represented by $U(m, \theta)$, where $m \in 0, 1, 2$ denotes the number of patents employed by the user to make a product, and $\theta$ represents the heterogeneity between users. For simplicity, it is assumed that the gross surplus does not depend on the combination of patents but on the number of patents, since the two patented technologies are symmetric. Further, we assume that users are distributed uniformly on the interval $[0, 1]$.

Our model specifies the user’s gross surplus function as the following quadratic form:

$$U(m, \theta) = \theta m + cm^2, \quad (m = 0, 1, 2)$$

where $c > -1/3$. The levels of gross surplus for each number of patents are $U(0, \theta) = 0$, $U(1, \theta) = \theta + c$, and $U(2, \theta) = 2\theta + 4c$. We find that a user with high $\theta$ obtains a high level of gross surplus for any number of patents used. The heterogeneity of gross surplus between

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6) See the Guidelines on the application of Article 81 of the EC Treaty for technology transfer agreements (2004/C101/02).
The initial differences in gross surplus are $\Delta U(1, \theta) = \theta + c$, and $\Delta U(2, \theta) = \theta + 3c$. The difference, $\Delta U(1, \theta)$, implies the user’s willingness to pay for the first patent (patent A or B) when a user does not have access to any patent, and $\Delta U(2, \theta)$ is the user’s willingness to pay for the second patent when a user already has access to the first patent. Note that the willingness to pay for an additional patent depends on $\theta$, and that a user with high $\theta$ obtains a high additional surplus. Lerner and Tirole (2004) assumes that the willingness to pay for an additional patent is the same between users. Under this assumption, a patent pool only offers single package license, since all users demand the same number of patents. In practice, however 12% of the patent pools in the Lerner et al. (2003) sample offered multiple package licenses. To explain the observed offering of multiple package licenses by 12% of the patent pools, we must allow for differences in the willingness to pay for an additional patent across users. Actually, due to variations in the ability or knowledge of each user, it is no wonder that there are differences in their benefits from using an additional patent.

The second difference in the gross surplus is $\Delta^2 U(2, \theta) = 2c$. When the two patents are technical substitutes (the two patents have some similarity), the willingness to pay for the second patent is smaller than that for the first patent. Then the value of $c$ is negative. When the two patents are technical complements, the willingness to pay for the second patent is not smaller than that for the first patent. Then the value of $c$ is not negative.

3. Patent pool pricing

3.1 Users’ decisions and the demand for package licenses

In this section, we characterize the equilibrium of the situation in which patents A and B are monopolistically licensed only by the patent pool formed by patent holders A and B. The patent pool could offer users two package licenses, A and AB. In package A, the pool offers a license for only patent A or patent B to users for the licensing fee $p_A$. In package AB, the pool offers licenses for both patent A and B to users for the licensing fee $p_{AB}$; the package AB is a bundled good. If a user only wants patent A or B, he/she can buy package A. If a user wants both patents A and B, he/she can buy the package AB. We also assume that a pool must pay nonnegative but very small costs for offering a package license. The pool does not offer the packages which any of users buy.

Given the licensing fees $p_A$ and $p_{AB}$, the users choose the number of patents $m$ to maximize their net surplus. When packages A and AB are offered by the patent pool, the type $\theta$ user chooses which package to buy in the following manner:

7) This specification satisfies the discrete form of what we refer to as the Spence-Mirrlees single-crossing condition in the Contract Theory literature; for example, Salanie (2002) and Bolton and Dewatripont (2005). This condition is known for separating different types of agents by offering larger allocations to higher types and making them pay for the privilege.
· The user does not buy any packages (the user choices $m = 0$), if $\theta + c - p_A < 0$.
· The user buys package A (the user choices $m = 1$), if $\theta + c - p_A \geq 0$ and $\theta + 3c < p_B$, where $p_B \equiv p_{AB} - p_A$.
· The user buys package AB (the user choices $m = 2$), if $\theta + 3c \geq p_B$.

Since the surpluses differ across different users, the number of patents chosen by each user also differs across users. Now we define $\theta_A \equiv p_A - c$, $\theta_B \equiv p_B - 3c$. The type $\theta_A$ user is indifferent between package A and not buying any package. And the type $\theta_B$ user is indifferent between package A and package AB. Each user behaves as follows:

· The users on interval $[\theta_A, \theta_B]$, where $\theta_A \equiv p_A - c$, do not buy any packages.
· The users on interval $[\theta_A, \theta_B]$, where $\theta_B \equiv p_B - 3c$, buy package A.
· The users on interval $[\theta_B, 1]$ buy package AB.

**Lemma 1** When given licensing fees that satisfy the inequality $p_B - p_A > 2c$, there are users who buy package A.

**Proof.** When $\theta_A < \theta_B$, there are users who buy package A. From the definition of $\theta_A$ and $\theta_B$, we can find $\theta_A < \theta_B \iff p_B - p_A > 2c$.

When licensing fees satisfy the inequality $p_B - p_A > 2c$, there are both users who buy package A and who buy package AB($\theta_A < \theta_B$). Then the demand for package A is

$$D_A = \theta_B - \theta_A.$$  \hfill (2)

The demand for package AB is

$$D_{AB} = 1 - \theta_B.$$  \hfill (3)

When licensing fees do not satisfy the inequality $p_B - p_A > 2c$, then no user buys package A ($D_A = 0$). The type $\theta$ user decides whether or not to buy package AB in the following manner:

· The user does not buy any packages (the user choices $m = 0$), if $2\theta + 4c < p_{AB}$.
· The user buys package AB (the user choices $m = 2$), if $2\theta + 4c \geq p_{AB}$.

Then, defining $\theta_C \equiv (p_{AB}/2) - 2c$, the demand for package AB is

$$D_{AB} = 1 - \theta_C.$$  \hfill (4)

### 3.2 Patent pool equilibrium

For simplicity, we ignore the costs paid by the pool members (the patent holders) for developing their own patents. Under the demand function of each package license, the patent pool’s profit is $\Pi_{pool} = p_A D_A + p_{AB} D_{AB} = p_A (D_A + D_{AB}) + p_B D_{AB}$. Note that $D_A + D_{AB}$ is the demand for only patent A or B. The patent pool decides $p_A$ and $p_B$ to maximize profit. From lemma 1 and the definition of demand (2), (3) and (4), profit is rewritten as follows:
The equilibrium licensing fees are given by the solution of the above profit maximization problem.

Now we consider the case where the patent pool offers both packages A and AB. The optimal licensing fees are the solution to the following profit maximization problem.

\[
\max_{p_A, p_B} p_A (1 - \theta_A) + p_B (1 - \theta_B),
\]

where \( p_B - p_A > 2c \). The interior solution should satisfy the following equations:

\[
\frac{\partial \Pi_{pool}}{\partial p_A} = (1 - p_A + c) - p_A = 0, \tag{7}
\]

\[
\frac{\partial \Pi_{pool}}{\partial p_B} = (1 - p_B + 3c) - p_B = 0. \tag{8}
\]

Since the second order condition is warranted, the licensing fees which satisfy (7) and (8) are \( p^*_A = (1 + c)/2 \), and \( p^*_B = (1 + 3c)/2 \). Checking that these licensing fees satisfy the inequality \( p_B - p_A > 2c \), we find that the optimal solution is \( p^*_A = (1 + c)/2 \) and \( p^*_B = (1 + 3c)/2 \) (that is \( p_{AB} = p_A + p_B = 1 + 2c \)) if \( c \) is negative.

Equations (7) and (8) are rewritten in the following forms:

\[
\Psi_A(p) = \Psi_B(p) = \Psi(p)
\]

Figure 1

\[
p_A = \frac{1 + c}{2}, \quad p_B = \frac{1 + 3c}{2}
\]
where $\Psi_A(p_A)$ and $\Psi_B(p_B)$ are the price elasticities of demand for patent A and patent B, respectively. Both $\Psi_A(p_A)$ and $\Psi_B(p_B)$ are increasing for each licensing fee. The licensing fees are determined such that $\Psi_A(p_A)$ and $\Psi_B(p_B)$ are equal to one. When $\Psi_A(p)$ is located above $\Psi_B(p)$ as in Figure 1, there are users who buy package A ($\theta_A < \theta_B$). In the case where $c$ has a negative value, patent B is not really attractive for users when compared with patent A. Then the price elasticity of demand for patent B is smaller than that for patent A: $\Psi_B(p) > \Psi_A(p)$. Similar to the mechanism of price discrimination, $p_A$ is higher than $p_B$ : $p_B = p_A - c < 0$.

If $c$ does not have a negative value, then the patent pool does not offer package A. The patent pool’s problem is

$$\max_{p_{AB}} (1 - \theta_c).$$

The optimal licensing fee satisfies

$$\frac{\partial \Pi_{pool}}{\partial p_{AB}} = (1 - \frac{1}{2}p_{AB} + 2c) - \frac{1}{2}p_{AB} = 0.$$ 

Since the second order condition is warranted, the licensing fee which satisfies (12) is $p_{AB}^* = 1 + 2c$. Then we get the following proposition.

**Proposition 1.** Patent pool pricing

1. **Multiple package licenses:** When the two patents are substitutes ($c < 0$), the patent pool offers two package licenses A and AB, and the licensing fees are, $p_A = (1 + c)/2$, $p_{AB} = 1 + 2c$ ($p_B^* = (1 + 3c)/2$).

2. **Single package license:** When the two patents are complements ($c \geq 0$), the patent pool only offers a single package AB, and the licensing fee is $p_{AB} = 1 + 2c$.

**Proposition 1** shows that patent pool pricing is characterized by $c$. When the technical relationship between patent A and B is that of substitutes ($c < 0$), both package license A, inclusive of only patent A or B, and package license AB, inclusive of both patent A and B, are offered by the patent pool. Package licensing fees are $p_A^* = (1 + c)/2$ and $p_{AB}^* = 1 + 2c$. In this case, there are both users who buy package license A and users who buy package license AB. The patent pool is willing to offer multiple package licenses by offering not only package license AB but also package license A in order to maximize profit.

On the other hand, when the technical relationship is complementary ($c \geq 0$), only package license AB, inclusive of both patent A and B, is offered by the patent pool. The package
licensing fee is \( p_{AB} = 1 + 2c \). In this case, there are users who buy only package license AB, if they buy the license. Since the technical relationship is complementary, there are no users who use only patent A or B. This case corresponds to patent pool pricing in Shapiro (2001) and binding demand margins in Lerner and Tirole (2004).

The patent pool only offers package AB, which is similar to a tie-in sale, and is not willing to offer multiple package licenses as doing so will not maximize profit. In order to avoid the tie-in sale, EU committee encourages patent pools to offer multiple package licenses. But our result is that none of the users buy package licenses when \( c \) is negative. Therefore, the recommendation of the competition authority makes no sense in the case \( c \geq 0 \) under our model.

4. Individual Pricing

4.1 Demand for each patent

In this section, we consider the case where a patent pool is not established. Patent holders A and B individually offer licenses to patent users. If a user wants to use patent A (patent B), the user must access patent holder A (patent holder B). If a user wants to use both patents, then the user must access both patent holders.

Given the licensing fee \( p_i \, (i = A, B) \), the users decide whether or not to choose each patent with the objective of maximizing their net surpluses. Since we assume that the two technologies are symmetric, the user buys the license with the a lower price when buying only one patent. We describe the users’ behavior as the follows.

For given \( p_A \) and \( p_B \), if there are users who buy only one license,
- The users on interval \([0, \theta_1]\), where \( \theta_1 \equiv \min\{p_A, p_B\} - c \), do not buy any licenses.
- The users on interval \([	heta_1, \theta_2]\), where \( \theta_2 \equiv \max\{p_A, p_B\} - 3c \), buy a license. When \( p_A < p_B \), the users buy license A. When \( p_A > p_B \), the users buy license B.
- The users on interval \([\theta_2, 1]\) buy both licenses A and B.

For given \( p_A \) and \( p_B \), if there are no users who buy only one technology,
- The users on interval \([0, \tilde{\theta}]\), where \( \tilde{\theta} \equiv (p_A + p_B)/2 - 2c \), do not buy any licenses.
- The users on interval \([\tilde{\theta}, 1]\) buy both licenses A and B.

Lemma 2. When the given licensing fees satisfy the inequality \(|p_B - p_A| > 2c\), there are users who buy only one license. If \( c < 0 \), there are always users who buy only one license.

Proof. When \( \theta_1 < \theta_2 \), there are users who buy only one license. From the definition of \( \theta_1 \) and \( \theta_2 \), we can find the following:

\[
\theta_1 < \theta_2 \iff \min\{p_A, p_B\} - c < \max\{p_A, p_B\} - 3c
\]

\[
\iff \max\{p_A, p_B\} - \min\{p_A, p_B\} > 2c
\]

\[
\iff |p_A - p_B| > 2c.
\]

If \( c < 0 \), \(|p_A - p_B| \geq 0 > 2c\) is satisfied.

From lemma 2, we can derive the demand functions of the users for patents \( i = A, B \), as
follows:

The case where $c < 0$ is

\[
\begin{align*}
  d_i(p_i, p_{-i}) &= 1 - \theta_1, \quad \text{if } p_i < p_{-i}, \\
  d_i(p_i, p_{-i}) &= \frac{1}{2}((1 - \theta_1) + (1 - \theta_2)), \quad \text{if } p_i = p_{-i}, \\
  d_i(p_i, p_{-i}) &= 1 - \theta_2, \quad \text{if } p_i > p_{-i},
\end{align*}
\]

(13) (14) (15)

where $\theta_1 = p_i - c$ and $\theta_2 = p_i - 3c$.

The case where $c \geq 0$ is

\[
\begin{align*}
  d_i(p_i, p_{-i}) &= 1 - \theta_1, \quad \text{if } |p_A - p_B| > 2c \text{ and } p_i < p_{-i}, \\
  d_i(p_i, p_{-i}) &= 1 - \bar{\theta}, \quad \text{if } |p_A - p_B| \leq 2c, \\
  d_i(p_i, p_{-i}) &= 1 - \theta_2, \quad \text{if } |p_A - p_B| > 2c \text{ and } p_i > p_{-i},
\end{align*}
\]

(16) (17) (18)

where $\bar{\theta} = (p_i + p_{-i})/2 - 2c$.

When $c$ is negative, the two patents are technically different but somewhat similar. There are always users who buy only one license. (13) is user demand for the patent with the lower licensing fee.

Users who want only one license buy the cheaper license since the two patents are symmetric, and users who want both licenses buy both the cheaper license and the more expensive license. (15) is user demand for the patent with the higher licensing fee. Users who want only one license do not buy the more expensive license. Only users who want both licenses buy the more expensive license, since these users can obtain a high additional surplus from the additional license. Therefore, the demand of (15) is smaller than that of (13). (14) is user demand for each patent when patent holders set the same licensing fee. Since the symmetric patents have the same licensing fee, users are indifferent between patents A or B. The user demand for each patent is half the sum of the demands from users who buy only one license and who buy both licenses.

When $c$ is non-negative, the two patents are technical complements. Users can obtain a higher additional surplus from the second license than the first license. But users do not buy the second license if the licensing fee of the second license is higher than the additional surplus from the second license. If the difference between the licensing fees of the two patents is large ($|p_A - p_B| > 2c$), the users only buy the license with the lower the licensing fee. Therefore, the user demand for the license is (16) if the licensing fee is set higher than that of the other license, and the user demand for the license is (18) if the licensing fee is set lower than the other license. On the other hand, if the difference between the licensing fees of the two patents is small ($|p_A - p_B| < 2c$), the user demand for the license is (17), since the users always buy both licenses.
4.2 Individual pricing equilibrium

The patent holders decide their licensing fees to maximizing their own profit, \( \pi_A = p_Ad_A \) or \( \pi_B = p_Bd_B \), given the other licensing fee. Each patent holder’s strategy is the licensing fee of the patent owned by the patent holder. The individual pricing equilibrium is characterized by the follows:

**Proposition 2. individual pricing \((c < 0)\)**

When the two patents are substitutes, Nash equilibrium licensing fees satisfy the following mixed strategy profile \( g^*(p) \):

\[
g^*(p) = \frac{1 + c - 2p}{-2cp} - \frac{1}{p} G^*(p), \quad p \in (0, \bar{p}],
\]

where \( \bar{p} = (1 + 3c)/2 \) and \( G^*(p) = \int_0^p g^*(p)dp \).

**Proof.** Given the other patent holder’s continuous density function \( g_{-i}(p) \), patent holder \( i \) decides \( g_i(p) \) to maximize the following expected profit subject to \( g_i(p) \geq 0 \) and \( \int_0^\infty g_i(p)dp = 1 \). We define the following Lagrange function:

\[
L = \int_0^\infty [1 - G_{-i}(p)] g_i(p) \pi_1(p)dp + \int_0^\infty G_{-i}(p) g_i(p) \pi_2(p)dp + \lambda [1 - \int_0^\infty g_i(p)dp],
\]

where \( \pi_1 \equiv p(1 + c - p) \) and \( \pi_2(p) = p(1 + 3c - p) \). The first order condition becomes

\[
[1 - G_{-i}(p)] \pi_1(p) + G_{-i}(p) \pi_2(p) = \lambda.
\]

Differentiating equation (21) with respect to \( p \) yields

\[
g_{-i}(p)[\pi_1(p) - \pi_2(p)] + G_{-i}(p)[\frac{d\pi_1(p)}{dp} - \frac{d\pi_2(p)}{dp}] - \frac{d\pi_1(p)}{dp} = 0.
\]

Equation (22) expresses a non-autonomous system of a first order, linear differential equation. Dividing (22) by \( d\pi_1(p)/dp - d\pi_2(p)/dp \),

\[
g_{-i}(p) = \beta(p) - \alpha(p) G_{-i}(p),
\]

where \( \alpha(p) \equiv \frac{d\pi_1(p)/dp - d\pi_2(p)/dp}{\pi_1(p)/p - \pi_2(p)/p} \) and \( \beta(p) \equiv \pi_1(p)/(\pi_1(p)/p - \pi_2(p)/p) \).

Note that the patent holder does not have an incentive to set the licensing fees higher than that of the other patent holder. Then the Nash equilibrium mixed strategy is the following:
where \( \bar{p} = (1 + 3c)/2 \).

When the two patents are substitutes, the patent holder attempts to set the licensing fee lower than the other patent holder in order to obtain more demand for the license (the patent holder attempts to obtain the demand (13)).

But when the licensing fee is too low, through the competition of the licensing fees between patent holders, the patent holder can achieve a better profit by setting a higher license fee than the other patent holder. The patent holder gives up obtaining the demand of users who buy only one license. Therefore, there does not exist a pure strategy Nash equilibrium, but a mixed strategy Nash equilibrium exists in this case. The mixed strategy Nash equilibrium is characterized in lemma 3:

**Lemma 3.** We characterize the mixed strategy profile \( g^*(p) \) in this equilibrium as follows:

1. \( \frac{dg^*(p)}{dp} < 0 \),
2. \( \lim_{p \to 0} g^*(p) = \infty \), \( g^*(\bar{p}) = 0 \).

**Proof.** Differentiate (19) with respect to \( p \),

\[
\frac{dg^*(p)}{dp} = \frac{1 + c + 2cG^*(p)}{2cp^2} - \frac{1}{p} g^*(p),
\]

Figure 2

![Graph showing the function g(p) with p on the x-axis and g(p) on the y-axis.](image)
where $G^*(p) \leq 1$ and $c > -1/3$. Since $G^*(p) \leq 1$ and $c > -1/3$, the following inequality is satisfied:

$$\frac{dg^*(p)}{dp} < \frac{1 + 3c}{2cp^2} - \frac{1}{p}g^*(p) < 0.$$  \hfill (26)

Therefore, we know that $dg^*(p)/dp$ is negative.

Next, we can check that $g^*(p)$ satisfies the following:

$$\lim_{p \to 0} g^*(p) = \frac{1}{c} \lim_{p \to 0} \left( \frac{1 + c}{2cp} \right) = \infty,$$

$$g^*(p) = -\frac{1 + 3c - 2\bar{p}}{2c\bar{p}} = 0.$$  \hfill (27)

Figure 2 shows the mixed strategy profile $g^*(p)$ in this equilibrium, where $\theta \in [0, 1], c = -0.2$. It implies that patent holders set the lower licensing fee.

When the two patents are complements ($c \geq 0$), Nash equilibrium is characterized as follows:

**Proposition 3. individual pricing ($c \geq 0$) When the two patents are complements, Nash equilibrium licensing fees are $p_A^* = p_B^* = 2(1 + 2c)/3$.**

**Proof.** We examine three candidates for the equilibrium.

Case 1. If the equilibrium satisfies $|p_A^* - p_B^*| > 2c$ and $p_A^* < p_B^*$,

* $p_A^* = \arg \max \{p_A(1 - \theta_1)\} = \frac{1 + c}{2}$, for any $p_B^*$ which satisfies the above inequality.

Then $p_A^* - p_B^* = c > 2c$ contradicts $c \geq 0$. Case 1 is not an equilibrium.

Case 2. If the equilibrium satisfies $|p_A^* - p_B^*| > 2c$ and $p_A^* > p_B^*$,

* $p_A^* = \arg \max \{p_A(1 - \theta_2)\} = \frac{1 + 3c}{2}$, for any $p_B^*$ which satisfies the above inequality.

Then $p_A^* - p_B^* = c > 2c$ contradicts $c \geq 0$. Case 2 is not an equilibrium.

Case 3. If the equilibrium satisfies $|p_A^* - p_B^*| \leq 2c$, then users always buy both licenses, when they buy the license. Given the other patent holders licensing fee, the patent holder sets the following licensing fee to maximize profit $p_i(1 - \bar{\theta})$. Patent holder $i$'s reaction function is written as follows:
Therefore, the Nash equilibrium is $p^*_A = p^*_B = (2 + 4c)/3$, since this does not contradict $|p^*_A - p^*_B| \leq 2c$.

When $c \geq 0$, the users buy both licenses, since the two patents are complements. It is not each licensing fee but the sum of the licensing fees that users are concerned with. Although patent holders attempt to set higher licensing fees to increase profit, demand from users for each license decreases.

5. Welfare Analysis

In sections 3 and 4, we characterize the equilibrium licensing fees with a patent pool and in the absence of a patent pool. Comparing the welfare of patent pool pricing with that of individual pricing, we analyze whether or not a patent pool enhances social welfare.

We define social welfare as the sum of users’ net surplus and patent holders’ profit. Since users’ payment for the patent is equal to the profit of patent holders A and B, social welfare is equal to the gross surplus of users. Therefore, the lower the licensing fee is, the greater social welfare is. Comparing patent pool pricing with individual pricing in equilibrium, we get the following proposition for social welfare.

**Proposition 4.** A patent pool reduces social welfare when the two patents are substitutes, but enhances social welfare when the two patents are complements.

**Proof.** Now, we consider the case where the two patents are substitutes ($c < 0$). From proposition 1-(1), we know that the patent pool offers two package licenses A and AB (multiple package licenses). The licensing fees are $p^*_A = (1 + 2c)/2$ and $p^*_{AB} = 1 + 2c$. We also know that the individual case has a mixed strategy Nash equilibrium and the licensing fee is probabilistic in the equilibrium from proposition 2. Although we cannot know the previously realized licensing fee, we know that the highest licensing fee is $\bar{p} = (1 + 3c)/2$, which brings the lowest social welfare. When users buy only one license, patent pool pricing is higher than individual pricing: $p^*_A - \bar{p} = -c/2 > 0$. When users buy two licenses, patent pool pricing is higher than individual pricing: $p^*_{AB} - 2\bar{p} = -c > 0$. We can find that a patent pool raises the licensing fee and reduces social welfare.

Next, we consider the case where the two patents are complements ($c \geq 0$). In this case, users always buy two patents whether we are considering the patent pool case or the individual case. From proposition 1-(2), the licensing fee for the two patents in the patent pool is $p^*_{AB} = 1 + 2c$. The licensing fee of the two patents for individual patent holders is $p^*_A + p^*_B = (4(1 + 2c))/3$. Comparing the licensing fees for the two cases, patent pool pricing is lower than individual pricing. Therefore, we can find that a patent pool reduces the licensing fee and enhances social welfare.

Next we investigate whether patent holders A and B have an incentive to form a patent pool. As the standard assumption of corporate merger theory, we assume that patent holders A and B
Proposition 5. Patent holders have an incentive to form a patent pool, regardless of the relationship between the two patents.

Proof. First, we consider the case where the two patents are substitutes \((c < 0)\). The profit of the patent pool is \(\Pi_{\text{pool}} = (1 + c)^2/4 + (1 + 3c)^2/4\). The profit of the individual case is probabilistic, since the strategies of patent holders are mixed strategies in equilibrium. In this case, it is when both patent holders set the licensing fee \(\bar{p}\) that both patent holders obtain the highest profit. Then the sum of both patent holder's profits is \(\pi_A + \pi_B = \bar{p}(1 - \theta_1) + \bar{p}(1 - \theta_2) = (1 + 3c)(1 + c)/2\). Since \(\Pi_{\text{pool}} - \pi_A - \pi_B = c^2 > 0\), we find that the patent holders have an incentive to form a patent pool, whenever the profit of patent holders are realized.

Next, we consider the case where the two patents are complements \((c \geq 0)\). The profit of the patent pool is \(\Pi_{\text{pool}} = (1 + 2c)^2/2\). The profit of the individual case is \(\pi_A + \pi_B = 4(1 + 2c)^2/9\). Therefore, we find that the patent holders have an incentive to form a patent pool, since \(\Pi_{\text{pool}} > \pi_A + \pi_B\).

Propositions 3 and 4 show that the effect of a patent pool on welfare and the patent holders' incentive to form a pool are characterized by the technical relationship between the patents. When the two patents are substitutes, there are the users who buy only the cheaper license. The patent holders compete severely on the licensing fee to obtain the demand of users who only purchase one license. In this case, the patent holder can increase profit by forming a patent pool and using its monopoly power as a cartel. Therefore, a patent pool reduces social welfare.

On the other hand, a patent pool increases social welfare when the two patents are complements. Shapiro (2001) indicates that the individual pricing of patent holders reduces the demand of users for the license, since the sum of the licensing fees required to maximize their profits rises. In this case, patent holders can reduce the licensing fee and increases their profit by forming a patent pool.

6. Conclusion

This paper investigates the anti-competitive effects of the patent pool which offers package licenses by allowing users to choose between different numbers of patents. The most important work of the paper is to consider the multiple package licenses that a patent pool offers to users. What are the characteristics of a patent pool that offers multiple package licenses? Is a patent pool that offers multiple package licenses efficient? We find that the technical relationship between patents in the pool characterizes patent holders' incentive to form a patent pool, the licensing behavior of the patent pool and social welfare. Our main results are as follows.

8) We do not focus on the process of patent pool formation and the stability of pools, as Brenner (2009) and some other studies have examined. In our paper, patent holders have an incentive to form a patent pool if the profit of the pool is larger than the sum of the patent holders' profits, since the equilibrium in the pool case is equal to the co-operative solution of the patent holders (the pool maximizes their joint profit).
1. If the patents included in the patent pool are technical substitutes, patent holders have an incentive to form a patent pool. Both a package license with only one patent and a package license with two patents are offered by the patent pool (multiple package licenses). Then, the form of the patent pool raises the licensing fee that users pay to patent holders. As result, the patent pool does not reduce social welfare.

2. If the patents included in the patent pool are technical complements, the patent holders have an incentive to form a patent pool. Only a package license that includes all patents is offered by the patent pool (the single package license). Then, the form of the patent pool reduces the licensing fee that users pay to patent holders. As a result, the patent pool enhances social welfare.

These results lead to five suggestions. First, from result (1), we suggest that patent pools that include only complementary patents are procompetitive; patent pools that include only substitute patents are anticompetitive. Similar to previous literature, such as Shapiro (2001) and Lerner and Tirole (2004), these results are consistent with current U.S. and European policies when patents with a complementary relationship are interpreted as essential patents.

Second, we suggest actually-observed patent pools may only include complementary patents; these patent pools may not include substitute patents. It is observed that about 88% of the patent pools surveyed by Lerner et al. (2003) offer a single package license. Our model concludes that the single package license is offered when the patent pool includes technical complements. Therefore, we can guess that many actually-observed patent pools may only include complementary patents, and that they therefore increase social welfare.

Third, we suggest that actually-observed patent pools offering multiple package licenses are suspected of including substitute patents. From our model, it is when patents in a pool are technical substitutes that multiple package licenses are offered by the patent pool. Therefore, we can guess that these patent pools may be anticompetitive.

Fourth, we suggest that patent holders have the incentive to form not only patent pools that include only complementary patents, but also to form patent pool that include both complementary and substitute patents. Therefore it is possible that anticompetitive patent pools are formed by patent holders if the competition authority does not pay much attention to the form of patent pools. Considering that many actually-observed patent pools offer single package licenses, that is to say, these patent pools do not include substitute patents, the competition authority's attention to patent pools may be sufficient.

Finally, it is less important in our model that the competition authorities, such as the European Commission etc, encourage patent pools to offer multiple package licenses. The licensing behavior of patent pools is determined by the profit maximization of the patent pools. If multiple package licenses increase the profit of a patent pool, the patent pool is willing to offer a multiple package license. Otherwise, the patent pool does not offer the multiple package license, as long as competition authorities do not enforce the patent pool to offer the multiple package licenses. Even if the competition authorities force patent pools that offer single package licenses to offer multiple package licenses, users do not buy the package...
licenses which include only a patent for the optimal licensing fee. As long as the competition authorities do not intervene the licensing fees of patent pools, it does not make sense for the competition authorities to force patent pools to offer the multiple package licenses.

In this paper, we find that patent pools offering multiple package licenses are not efficient. But if our model adopts a multi-dimensional user's type space, that is our model considers the variety of user using patents, it is possible that the patent pool offering multiple package licenses is efficient. This topic is left for further research.

REFERENCES


