Innovation and Manufacturing Offshoring with Fully Endogenous Productivity Growth

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Abstract

This paper investigates the relationship between net offshoring patterns for innovation and manufacturing and fully endogenous productivity growth in a two-country model. The occupational choice of skill-differentiated workers into low-skilled employment in production and high-skilled employment in innovation determines labor market allocations, and perfect investment mobility allows firms to shift innovation and manufacturing independently between countries. These mechanisms generate a tension between access to technical knowledge and low-cost high-skilled labor in the location decision for innovation, which results in innovation and manufacturing tending to concentrate in the asset-wealthy (asset-poor) country when trade costs are high (low). The model exhibits a positive relationship between innovation costs and the concentration of industry and innovation, ensuring that a rise in knowledge diffusion between countries coincides with increases in net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country, and a faster rate of productivity growth, when the asset-wealthy country has larger shares of innovation and production.

**JEL Classifications:** F12, F43, R11

**Keywords:** innovation offshoring, manufacturing offshoring, endogenous productivity growth, process innovation, industry location, international trade

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1 Introduction

Although research and development (R&D) has traditionally concentrated in advanced countries, in recent years firms have begun to shift innovation offshore to emerging economies. For example, despite the steady position of the United States (US) as the largest producer of R&D services in terms of gross domestic expenditure (NSF 2016), over the past decade its trade balance in R&D services has deteriorated, even shifting to a deficit for several years, as shown in Figure 1.\(^1\) This trend appears to be driven by intra-firm trade between US parent firms and their foreign affiliates, and by growth in the US trade deficit in R&D services with emerging economies such as China and India. On average, trade deficits with China and India grew at rates of 26% and 25% between 2008 and 2014 to become US$2.1 billion and US$2.2 billion (BEA 2015).\(^2\)

There is a growing empirical literature investigating the links between R&D offshoring, innovation performance, and economic growth. At the firm level, Nieto and Rodríguez (2011) and Bertrand and Mol (2013) find that offshoring R&D leads to

\(^1\)Annual growth rates in US imports and exports of R&D services averaged 17% and 12% between 1999 and 2013 (BEA 2015).

\(^2\)The main hosts for R&D offshoring outside the US and the EU are Brazil, China, India, Russia, Singapore, and Taiwan (Hausmann et al. 2007; Puga and Trefler 2010; Santos-Paulino et al. 2014).

![Figure 1: U.S. Net Exports in R&D Services (BEA 2015)](image-url)
a higher propensity for the introduction of new products. Similarly, Rodríguez and Nieto (2016) document a positive relationship between innovation offshoring and sales growth. At the aggregate level, D’Agostino et al. (2013) show that OECD regions with firms that offshore innovation to emerging economies have more patent applications, and Castellani and Peiri (2013) report that European regions with a greater number of outward oriented R&D investment projects exhibit higher growth rates for labor productivity. While the empirical literature suggests a positive relationship between R&D offshoring and innovation-based economic growth, to the best of our knowledge this relationship has not been modelled formally.

This paper develops an endogenous market structure and endogenous growth framework (Peretto 1996; Aghion and Howitt 1998; Peretto and Connolly 2007; Etro 2009) to study the relationship between offshoring patterns in innovation and manufacturing and productivity growth. In particular, we extend the two-country model of Davis and Hashimoto (2015) to include an occupational choice for skill-differentiated workers between low-skilled employment in production and high-skilled employment in R&D. Firms produce differentiated products for supply to domestic and export markets, and invest in process innovation to reduce future production costs. The free movement of investment allows firms to shift innovation and production separately between countries with the aim of minimizing costs (Martin and Ottaviano 2001).

The framework captures two factors that have been emphasized in the business literature when considering the attractiveness of a location for R&D: access to technical knowledge and the supply of low-cost labor (see, for example, Chung and Yeaple 2008; Manning et al. 2008; Lewin et al. 2009; Demirbag and Glaister 2010). On the one hand, knowledge spillovers from production to innovation are local in nature, leading to greater spillovers in the country hosting a larger share of industry. On the other hand, the concentration of industry generates greater demand for high-skilled labor in innovation, pushing up high-skilled wages. These factors generate a tension in the
firm-level location decision for innovation.

At the aggregate level there is a positive circular causality between the location patterns of innovation and production. A higher national share of production strengthens knowledge spillovers and attracts R&D activity. In turn greater high-skilled employment raises income, increases market size and attracts production. Considering countries with symmetric labor forces, but different levels of asset wealth, we show that the circular causality results in the concentration of economic activity in the asset-wealthy country for high trade costs and in the asset-poor country for low trade costs.\(^3\)

These location patterns lead to three cases for the direction of net offshoring flows. For high trade costs, the market of the asset-wealthy country is not large enough to support the innovation and production of all domestically-owned firms, and net offshoring flows towards the asset-poor country. For intermediate trade costs, however, the asset-wealthy has a large market that allows it to maintain large shares of innovation and production, and it therefore receives net offshoring inflows from the asset-poor country. Lastly, for low trade costs, net offshoring flows towards the larger market of the asset-poor country.

An important feature of the framework is a positive relationship between the unit cost of process innovation and the geographic concentration of industry, as the benefit of greater knowledge spillovers is offset by the cost of rising high-skilled wages. An increase in the unit cost of process innovation reduces optimal firm-level employment in innovation, resulting in lower overall per-period labor costs and higher per-period profits, and attracting new firms into the market. As such, greater industry concentration coincides with a higher level of market entry and slower rate of productivity growth.

Focusing on the case for which the asset-wealthy country has greater shares of in-

\(^3\)Ekholm and Hakkala (2007) develop a static general equilibrium model in which production requires an intermediate R&D input that can be located separately from production, and show that various location patterns are possible depending on the level of trade costs.
novation and production, we investigate the effects of an improvement in international knowledge diffusion, and find that net offshoring flows from the asset-wealthy country to the asset-poor country rise as firms offshore innovation to the asset-poor country to take advantage of lower high-skilled wages. As a consequence, the increased dispersion of innovation and production away from the asset-wealthy country leads to lower unit costs for process innovation and a higher rate of productivity growth.

Our paper is closely related to the international trade literature that builds on the two-country variety-expansion model of Grossman and Helpman (1991) to consider the effects of manufacturing offshoring on innovation-based growth (Martin and Ottaviano 1999, 2001; Gao 2005, 2007; Naghavi and Ottaviano 2009a; 2009b). Within the models of this literature, it is common for innovation to agglomerate fully in the advanced country either by assumption or as a result of localized knowledge spillovers. As such, these models are not viable for the study of how R&D offshoring affects economic growth. Davis (2013) attempts to investigate the trade-off between localized knowledge spillovers and high-skilled wage costs in the R&D location choice for firms by extending Martin and Ottaviano (1999) to include an occupational choice for skill-differentiated workers. The link between industry location and economic growth is cut, however, leaving no relationship between economic growth and offshoring patterns. Thus, the key contribution of our paper is the introduction of a framework that allows for a theoretical study of the positive link between R&D offshoring and innovation-based economic growth that has been documented in the empirical literature.

The outline of the paper is as follows. Section 2 introduces the model and Section 3 characterizes industry and innovation location patterns. In Section 4 we investigate the directions of net offshoring flows, and in Section 5 we consider the relationships between offshoring patterns, market entry, and productivity growth. Section 6 concludes.
2 The Model

Two countries, home and foreign, potentially employ labor in four activities: traditional production, manufacturing, process innovation, and market entry. Home and foreign have symmetric labor endowments, and although there is no international migration, skill-differentiated workers choose between low-skilled employment in production and high-skilled employment in innovation within each country. We focus on the home country while introducing the model. The setup for foreign is analogous, however, with an asterisk denoting variables associated with foreign.

2.1 Household Preferences

Dynastic households in each country select optimal expenditure-saving paths with the aim of maximizing utility over an infinite time horizon. The lifetime utility of the representative household in the home country is

\[ U(t) = \int_t^\infty e^{-\rho(t-\tau)} \left( \alpha \ln C_X(\tau) + (1 - \alpha) \ln C_Y(\tau) \right) d\tau, \]

where the consumptions of a composite of manufacturing goods and a traditional good are \( C_X \) and \( C_Y \), the subjective discount rate is \( \rho \), and \( \alpha \in (0, 1) \). The manufacturing composite is

\[ C_X(t) = \left( \int_0^{N(t)} c_i(t)^{\frac{\sigma-1}{\sigma}} di + \int_0^{N^*(t)} c_j(t)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \]

where \( c_i \) and \( c_j \) are the demands for manufacturing varieties \( i \) and \( j \) of the \( N \) and \( N^* \) varieties produced in home and foreign, and \( \sigma > 1 \) is the elasticity of substitution. Lifetime utility is maximized subject to a national flow budget constraint:

\[ \dot{B}(t) = r(t)B(t) + I(t) - E(t), \]
where $E$ is household expenditure, $I$ is labor income, $B$ is asset wealth, $r$ is the interest rate, and a dot over a variable denotes time differentiation. The solution to the household’s utility maximization problem is the optimal expenditure-saving path described by the Euler condition: $\dot{E}/E = \dot{E}^*/E^* = r - \rho$, with equal access to an international financial market ensuring a common interest rate on asset wealth, and common motions for household expenditure. To simplify notation, we suppress time arguments when possible.

With identical investment opportunities, the asset wealths of home and foreign accumulate at the same rate $(\dot{B}/B = \dot{B}^*/B^*)$, and national shares of asset wealth are constant across time. Summing across the flow budget constraints (3) to obtain world expenditure $E_W \equiv E + E^*$ as a function of world asset wealth $B_W \equiv B + B^*$, and using the result to substitute the net return to investment $(r - \dot{B}_W/B_W)$ out of the national flow budget constraint yields home expenditure as follows:

$$E = I + b(E_W - I_W),$$

with $I_W \equiv I + I^*$ and national shares of asset wealth determined by initial levels of asset wealth: $b \equiv B/B_W$ and $b^* \equiv B^*/B_W$.

At each moment in time, households allocate constant shares of expenditure to the purchase of the manufacturing composite and the traditional good: $P_X C_X = \alpha E$ and $P_Y C_Y = (1 - \alpha)E$, where $P_X$ is a price index for manufacturing goods and $P_Y$ is the traditional good price in home. The price index over manufacturing goods is

$$P_X = \left(\int_0^N p_{X_i}^{1-\sigma} di + \int_0^{N^*} (\zeta p_{X^*_j}^{*})^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}},$$

where $p_{X_i}$ and $p_{X^*_j}^*$ are the prices of goods produced in home and foreign. Iceberg trade costs are incurred on international shipments with a shipment of $\zeta > 1$ units required for every unit sold in an export market. Viewing (5) as the unit expenditure function
over manufacturing goods, Shephard’s Lemma yields the home country demands for home and foreign produced varieties:

\[ c_i = \alpha p_{X_i}^{\sigma} P_X^{\sigma-1} E, \quad c_j = \alpha (\zeta p_{X_j})^{-\sigma} P_X^{\sigma-1} E. \]  

The demand conditions for foreign households are analogous.

### 2.2 Occupational Choice

Home and foreign have equal masses of workers \((Z = Z^*)\) with heterogeneous skill levels \(z\) that follow continuous uniform distributions with support \([0, 1]\). Workers are free to choose between employment in innovation or production. A worker employed in production supplies one unit of low-skilled labor, regardless of skill level, earning the low-skilled wage rate \(w_L\). A worker employed in innovation supplies \(z\) units of high-skilled labor, earning income \(zw_H\), where \(w_H\) is the high-skilled wage rate.

National labor markets are competitive with all firms paying the same low-skilled and high-skilled wages. When there is positive employment in production and innovation, one marginal worker potentially earns the same incomes from low-skilled and high-skilled employment, and is therefore indifferent between employment type. The skill level of this marginal worker equals the relative wage rate \(z = \omega = w_L/w_H\), and separates the labor force into workers with skill levels \(z \in [0, \omega]\) who choose employment in production, and workers with skill levels \(z \in [\omega, 1]\) who choose employment in innovation. This national labor allocation results in the following effective low-skilled and high-skilled labor supplies for the home country: \(L = \omega Z\) and \(H = (1-\omega^2)Z/2\). Expected national labor income, conditional on employment levels, is therefore \(I(\omega) = w_L L + w_H H = w_L (1+\omega^2)Z/(2\omega)\).
2.3 Traditional Production

Traditional firms employ one unit of low-skilled labor with a constant returns to scale technology to produce one unit of output for supply to an international market characterized by free trade. Setting low-skilled labor as the model numeraire, we assume that the world demand for traditional goods is large enough that both countries produce traditional goods at all moments in time. The traditional good price and low-skilled wage rates then equalize across countries, \( P_Y = P_Y^* = w_L = w_L^* = 1 \), and the home and foreign demands for traditional goods determine the world demand for low-skilled labor in traditional production:

\[
L_Y + L_Y^* = (1 - \alpha)E_W. \tag{7}
\]

2.4 Manufacturing

The manufacturing sector is monopolistically competitive (Dixit and Stiglitz 1977), with each firm producing a single unique product for supply to domestic and export markets. In addition to production, each firm employs high-skilled labor \((h_I)\) in process innovation and low-skilled labor \((l_F)\) in firm management. The production technology of firm \(i\) with production located in home is

\[
x_i = \theta^\gamma l_{Xi}, \tag{8}
\]

where \(x_i\) and \(l_{Xi}\) are firm-level output and low-skilled employment in production, \(\theta\) is firm-level productivity, and \(\gamma \in (0, 1)\) is the productivity elasticity of output. Although each firm employs a unique production technology, we assume that productivity levels are symmetric across firms \((\theta = \theta^*)\), regardless of the location of production.

Firms maximize operating profit \((\pi = px - l_X)\) by setting price equal to a constant markup over unit cost: \(p_X = p_X^* = \sigma/((\sigma - 1)\theta\gamma)\), where we now suppress the firm
index $i$. Equating supply with the demands from home and foreign households (6),
\[ x = c + \zeta c^* , \]
onoptimal operating profit for a firm with production located in home is
\[ \pi = \frac{l_X}{\sigma - 1} = \frac{\alpha p_X^{1-\sigma}}{\sigma} \left( \frac{E}{P_X^{1-\sigma}} + \frac{\varphi E^*}{P_X^{1-\sigma}} \right), \tag{9} \]
where $\varphi \equiv \zeta^{1-\sigma} \in (0, 1)$ describes the freeness of trade between countries with $\varphi = 0$ indicating prohibitively high trade costs and $\varphi = 1$ indicating perfectly free trade.

Firms are free to shift production between countries, with the aim of maximizing operating profit (Martin and Ottaviano 2001). Therefore, when there are active manufacturing sectors in both countries, operating profit is the same for all firms, regardless of production location; that is, $\pi = \pi^*$. Combining the price indices (5) with operating profit (9), we solve for the share of firms locating production in home as
\[ s_X \equiv \frac{N}{N_W} = \frac{E - \varphi E^*}{(1 - \varphi)E_W} = \frac{b - \varphi b^*}{1 - \varphi} + \frac{(1 + \varphi)(b^* I - bI^*)}{(1 - \varphi)E_W}, \tag{10} \]
where $N_W \equiv N + N^*$. Substituting the manufacturing shares of each country back into operating profit yields firm-level employment in production for all locations:
\[ l_X = \frac{\alpha(\sigma - 1)E_W}{\sigma N_W}, \tag{11} \]
where we have used the production function (8).

### 2.5 Process Innovation

Firms invest in process innovation with the aim of lowering unit production costs. The evolution of productivity for a firm with process innovation located in home follows
\[ \dot{\theta} = k\theta h_I, \tag{12} \]
where \( h_I \) is firm-level high-skilled employment and \( k\theta \) is labor productivity in process innovation. Following the in-house process innovation literature (Smulders and van de Klundert 1995; Peretto 1996; Peretto and Connolly 2007), firm-level R&D exhibits an intertemporal knowledge spillover through which the technical knowledge created by current innovation activity improves the labor productivity of future innovation efforts. Specifically, the productivity coefficient \( \theta \) represents the current stock of technical knowledge, and \( k \) determines the strength of intertemporal knowledge spillovers from the stock of knowledge into current innovation activity.

There is a large body of empirical research documenting both the localized nature of knowledge spillovers and the international scope for knowledge diffusion (Bottazzi and Peri 2007; Mancusi 2008; Coe et al. 2009; Ang and Madsen 2013).\(^4\) Adapting the setup of Baldwin and Forslid (2000), we capture the geographic nature of knowledge diffusion with the following specification for the strength of knowledge spillovers from production into process innovation located in home:

\[
k = s_X + \delta(1 - s_X),
\]

where the localized nature of knowledge spillovers is regulated by the degree of knowledge diffusion \( \delta \in (0, 1) \). Therefore, the labor productivity of high-skilled workers in innovation is determined as the weighted-average productivity of the observable stock of knowledge, with a stronger weighting for production technologies employed in proximity to the innovation department of the firm.

Accounting for process innovation costs \( (w_H h_I) \) and fixed per-period management costs \( (l_F) \), total per-period profit is \( \Pi = \pi - w_H h_I - l_F \) and firm value equals the

\(^4\)See Keller (2004) for a survey of the various channels through which knowledge spillovers arise.
presented discounted value of the expected future profit stream:

\[ V(t) = \int_t^\infty e^{-\int_t^\tau (r(\tau') + \lambda)d\tau'} \Pi(\tau)d\tau', \]  

(14)

where \( \lambda > 0 \) is an instantaneous default rate that indicates the probability that a firm-specific shock forces the firm to exit the market (Baldwin 1999).

Each firm sets its level of high-skilled employment in process innovation to maximize firm value subject to (12). We solve this optimization problem with the following current-value Hamiltonian function: \( F = \Pi + p_I kH_I \), where \( p_I \) is the internal price of a unit mass of new innovations developed by a firm in home over the time interval \( dt \), and each firm perceives the price indices (5) as constant when considering the impact of changes in its price on profits, given its small market share. The first order conditions for optimization provide the following static and dynamic efficiency conditions:

\[ p_I = \frac{w_H}{k\theta}, \quad p_I(r + \lambda) - \dot{p}_I = \frac{\partial \pi}{\partial \theta} = \frac{(\sigma - 1)\gamma\pi}{\theta}. \]  

(15)

The internal price of process innovation captures two key factors in the location decision for R&D that have been emphasized in the literature: the observable stock of technical knowledge and the cost of employing high-skilled labor (Chung and Yeaple 2008; Manning et al. 2008; Lewin et al. 2009; Demirbag and Glaister 2010).

Firms are free to shift their innovation actives between countries, ensuring a common internal price for new process innovations when there is innovation located in both home and foreign; that is, \( p_I = p_I^* \). We combine this condition with \( \omega = 1/w_H \) to obtain the home production share required to equate the price of new process innovations across countries:

\[ s_X = \frac{w_H - \delta w_H^*}{(1 - \delta)(w_H + w_H^*)} = \frac{\omega^* - \delta\omega}{(1 - \delta)(\omega + \omega^*)}, \]  

(16)
where we have used (13) and (15). Similar to the framework of Ekholm and Hakkala (2007), the degree of knowledge diffusion regulates the range of relative high-skilled wages over which dispersed location patterns are feasible; that is, \( w_H/w_H^* \in (\delta, 1/\delta) \) is required for \( s_X \in (0, 1) \). Substituting (16) back into (13) yields the equilibrium strength of knowledge spillovers into innovation in home: \( k = (1 + \delta)\omega^*/(\omega + \omega^*) \).

### 2.6 Market Entry

In order to focus on firm-level investment in process innovation, we consider a simple setup for market entry with new firms employing low-skilled labor as they prepare to enter the manufacturing industry. Following Etro (2004), Peretto and Connelly (2007), and Peretto and Valente (2015), we assume that the cost of market entry is proportionate to the value of production after market entry, generating a positive relationship between entry costs and market size. Specifically, the costs of market entry in home and foreign are \( V = p_X x \) and \( V^* = p_X^* x^* \), and are equal when both countries have positive shares of manufacturing, since \( p_X x = p_X^* x^* \).

The value of a new firm equals the present value of the future profit stream earned after entering the manufacturing sector (14). Free entry drives firm value down to the cost of market entry:

\[
V = p_X x = \frac{\alpha E_W}{N_W}, \tag{17}
\]

where we have used (9) and (11). The time derivative of (14) combined with (17) yields the following no-arbitrage condition for investment in market entry (Grossman and Helpman 1991):

\[
(r + \lambda)V - \dot{V} = \pi - w_H h_I - l_F. \tag{18}
\]

The investment conditions (15) and (18) imply that \( \omega k = \omega^* k^* \), \( h_I/\omega = h_I^*/\omega^* \), and
\[ k h_I = k^* h_I^* \] when firms locate innovation in home and foreign.

Aggregating across countries, we obtain the following differential equation to describe the dynamics of market entry and exit:

\[
\dot{N}_W = \frac{L_N + L_N^*}{p_X} - \lambda N_W = \frac{(L_N + L_N^*) N_W}{\alpha E_W} - \lambda N_W
\]

(19)

where we have used (8), (9) and (11), and \( L_N \) and \( L_N^* \) are the national levels of low-skilled labor employed in market entry. At each moment in time \( \lambda N_W \) firms default and are forced to exit the market.

3 Long-run Location Patterns

We now derive the long-run industry and innovation location patterns consistent with equilibrium in the labor and investment markets. In order to simplify the analysis we consider the level of market entry relative to market size: \( n \equiv N_W / (\alpha E_W) \). In addition, without loss of generality, we assume that the home country has a greater share of asset wealth \( (b \geq 1/2) \) for the remainder of the paper.

3.1 National Labor Allocations

We first solve for a condition that determines the national labor allocations associated with common prices for goods \( p_X = p_X^* \) and innovations \( p_I = p_I^* \). Using the flow budget constraint for total expenditure (3) with \( B_W = N_W V = \alpha E_W \) and \( \dot{E}_W / E_W = r - \rho \), we find that total labor income is determined proportionately with total expenditure:

\[ I_W = (1 - \alpha \rho) E_W. \]

Then, equating (10) and (16) and reorganizing the result yields

\[
\frac{\alpha \rho (b - \varphi b^*)}{1 - \varphi} + \frac{(1 - \alpha \rho)(I - \varphi I^*)}{(1 - \varphi) I_W} = \frac{\omega^* - \delta \omega}{(1 - \delta)(\omega + \omega^*)},
\]

(20)
which we refer to as the *share locus*, as it indicates the relative wage combinations consistent with national labor allocations and production shares that ensure equalized operating profits and innovation costs across countries at all moments in time.

Second, we derive the dynamics associated with national labor allocations. Common motions for household expenditures ($\dot{E}/E = \dot{E}/E^*$) result in constant production shares ($\dot{s}_X = 0$) and common motions for relative wages ($\dot{\omega}/\omega = \dot{\omega}/\omega^*$). Combining (15) with $I_W = (1 - \alpha \rho)E_W$, $h_I/\omega = (H/\omega + H^*/\omega^*)/N_W$, and $\dot{E}_W/E_W = -((H/\omega + H^*/\omega^*)/I_W)(\dot{\omega}/\omega) = r - \rho$, we obtain the following motion for the evolution of relative wages:

$$\frac{\dot{\omega}}{\omega} = \frac{(n_H - n)(\rho + \lambda)I_W}{(L + L^*)n}, \quad n_H = \frac{(\sigma - 1)\gamma\omega k}{\sigma(\rho + \lambda)} - \frac{(1 - \alpha \rho)(H/\omega + H^*/\omega^*)\omega k}{\alpha(\rho + \lambda)I_W}, \quad (21)$$

where $n_H$ is the steady-state level of market entry consistent with optimal investment in process innovation and equilibrium in the high-skilled labor market.

Third, we derive an expression to describe the evolution of market entry. Substituting (7), (11), and (19) with $I_W = (1 - \alpha \rho)E_W$ into the low-skilled labor market clearing condition $L + L^* = L_Y + L^*_Y + N_Wl_X + N_Wl_F + L_N + L^*_N$, we have

$$\frac{\dot{n}}{n} = (n_L - n)l_F + \frac{(H/\omega + H^*/\omega^*)}{I_W} \frac{\dot{\omega}}{\omega}, \quad n_L = \frac{(1 - \alpha \rho)(L + L^*)}{\alpha l_FL_W} - \frac{\sigma - \alpha}{\alpha \sigma l_F} - \frac{\lambda}{l_F}, \quad (22)$$

where $n_L$ is the steady-state level of market entry consistent with optimal investment in market entry and equilibrium in the low-skilled labor market. Equating (21) and (22) yields a second steady-state condition ($\dot{\omega} = \dot{n} = 0$) for national labor market allocations that we refer to as the *investment locus*: $n_H = n_L$.

Together the investment and share loci determine long-run labor allocations. The investment locus is depicted by curve $aa$ in Figure 2 and has a strictly negative slope (see Appendix A). As illustrated by the curve $bb$, the share locus has two hyperbolic branches with asymptotes implicitly defined by $\omega = \omega^*$ and $(\delta - \delta \varphi + (1 - \delta \varphi)\omega^* + (1 - \delta \varphi)\omega^*).$
These stylized labor allocation patterns can be reproduced numerically using $\alpha = 0.95$, $\sigma = 2.5$, $\gamma = 0.5$, $\rho = 0.01$, $\lambda = 0.01$, $l_F = 0.05$, $b = 0.75$, $\delta = 0.45$, and $\varphi_B = 0.89$, for both panels, and with $\varphi = 0.75$ for Panel (a) and $\varphi = 0.95$ for Panel (b).

The long-run equilibria associated with the intersection between the investment locus and the upper and lower branches of the share locus are shown in Figures 2a and 2b. In Appendix A, we investigate the local dynamics of the model and derive two sufficient conditions for the saddle-path stability of a long-run equilibrium with dispersed industry and innovation: first that $\omega k l_F > \rho + \lambda$ and second that the share locus have a greater slope than the investment locus.\(^6\) We limit our analysis to steady states that satisfy these conditions.

\(^5\)Using the expressions introduced in Appendix A, the share locus can be rewritten as follows:

$$\frac{\omega(1 + \omega^*)(b - b^*)(1 - \delta)(1 + \varphi)\alpha \rho}{(\omega^* - \omega)(\delta - \varphi + (1 - \delta \varphi)\omega \omega^* + (1 - \delta)(1 + \varphi)(b^* - b \omega^*)\alpha \rho)} = 1.$$ 

The denominator describes the asymptotes that arise for $b > b^*$.  

\(^6\)Numerical simulations suggest that these sufficient conditions are actually necessary conditions for a saddle-path stable equilibrium, although we are unable to show this result analytically.
3.2 Industry and Innovation Location Patterns

National shares of industry are described by (16). In Figure 2, the full concentration of industry occurs along the \( \omega^*/\omega = 1/\delta \) dashed line \( (s_X = 1) \), and in foreign along the \( \omega^*/\omega = \delta \) dashed line \( (s_X = 0) \). Similarly, denoting the numbers of firms locating process innovation in home and foreign by \( m = H/h_I \) and \( m^* = H^*/h^*_I \), with high-skilled labor employed solely in process innovation, the home share of innovation output is

\[
s_I \equiv \frac{m \theta}{m \theta + m^* \theta^*} = \frac{w_H H}{w_H H + w^*_H H^*} = \frac{(1 - \omega^2)\omega^*}{(1 - \omega^2)\omega^* + (1 - \omega^2)\omega},
\]

(23)

where we have used \( kh_I = k^* h^*_I \) and \( h_I/\omega = h^*_I/\omega^* \). National shares of innovation activity are described by the investment locus in Figure 2, with innovation fully concentrated in home at point \( a (s_I = 1) \), and innovation fully concentrated in foreign at point \( a^* (s_I = 0) \). The shaded areas indicate national labor allocations that are not consistent with labor market equilibrium.

We characterize industry and innovation location patterns according to the level of trade costs, and obtain the following proposition.

**Proposition 1**

(i) If \( \varphi < \varphi_B \), the asset-wealthy country has larger shares of industry and innovation, with the full concentration of industry for \( \varphi \in (\varphi_X, \varphi_B) \) and the full concentration of innovation for \( \varphi \in (\varphi_I, \varphi_B) \). (ii) If \( \varphi > \varphi_B \), the asset-poor country has larger shares of industry and innovation, with the full concentration of industry for \( \varphi \in (\varphi_B, \varphi_X) \) and the full concentration of innovation for \( \varphi \in (\varphi_B, \varphi^*_I) \), where

\[
\varphi_B \equiv \frac{\delta + \omega \omega^* + (1 - \delta)(b^* - \omega^* b)\alpha \rho}{1 + \delta \omega \omega^* - (1 - \delta)(b^* - \omega^* b)\alpha \rho},
\]

with positive values for both the numerator and denominator since \( 1 > \alpha \rho \). Note that \( \varphi_I < \varphi_X \) and \( \varphi^*_I > \varphi^*_X \) for \( \delta < \delta \), but \( \varphi_I \geq \varphi_X \) and \( \varphi^*_I \leq \varphi^*_X \) for \( \delta \geq \delta \).

Proof: See Appendix B.
There is a positive circular causality between the location patterns of production and process innovation. A higher share of production strengthens knowledge spillovers and attracts innovation, which in turn raises high-skilled labor income, expands market size, and attracts production. With changes in national shares of asset wealth, however, the initial direction of this circular causality turns on the balance of the three effects captured by the share locus (Davis 2013). The investment income effect is described by the first term on the LHS of (20), with an increase in relative asset wealth expanding a country’s relative market size. The labor income effect is represented by the second term on the LHS of (20), with a rise in high-skilled labor income also increasing national market size. The investment and labor income effects link national shares of industry with market size through the home market effect (Krugman 1980), the strength of which is increasing in the freeness of trade. The innovation cost effect is described by the RHS of (20), and regulates how fast high-skilled wages can rise as a national market expands before firms begin to relocate process innovation internationally to reduce the cost of employing high-skilled labor. The innovation cost effect is increasing in knowledge diffusion: stronger knowledge spillovers facilitate a faster rate of high-skilled wage growth.

Consider the national labor market adjustments that occur when a small increase in $b$ expands the home share of production through the investment income effect. Holding the degree of knowledge diffusion constant, we use the freeness of trade to identify two cases for the direction of labor market adjustments as the economy returns to equilibrium. First, for $\varphi < \varphi_B$, the innovation cost effect dominates the labor income effect, and the initial shift in location patterns leads to a lower cost for innovation in home: $p_I < p_I^*$. Firms then relocate innovation and production to the home country, with $w_H/w_H^*$ increasing at a faster rate than $k/k^*$ until process innovation costs are equal again. In this case, the asset-wealthy country becomes the net exporter of manufacturing goods and process innovations, and has a larger market with greater
high-skilled employment. Second, for $\varphi > \varphi_B$, the labor income effect dominates, and the initial shift in location patterns generates higher innovation costs for home: $p_I > p^*_I$. As a result, innovation and production relocate to foreign with $k/k^*$ decreasing at a faster rate than $w_H/w^*_H$ until $p_I = p^*_I$ again. In this case, the asset-poor country becomes the net exporter of manufacturing goods and process innovations, and has a larger market with greater high-skilled employment.

The degree of knowledge diffusion determines whether the full concentration of production coincides with the full concentration of innovation. If $\delta < \delta^*$, the knowledge spillover advantage of the larger national market discourages firms from locating innovation in the smaller country, and innovation concentrates fully before industry; that is, $\varphi_X < \varphi_I$ and $\varphi^*_I > \varphi^*_X$. In contrast, if $\delta > \delta^*$, a high degree of knowledge reduces the localized benefits of knowledge spillovers, allowing the smaller country to attract innovation with its low-cost high-skilled labor, even with industry fully concentrated in the larger country; that is, $\varphi_X > \varphi_I$ and $\varphi^*_I < \varphi^*_X$.

### 3.3 Offshoring Patterns

The direction of net offshoring flows in the manufacturing sector is determined as the difference between national shares of asset wealth and production (Martin and Ottaviano 1999). For the home country we have

$$ S_X \equiv b - s = \frac{(b + \delta b^*)\omega - (b^* + \delta b)\omega^*}{(1 - \delta)(\omega + \omega^*)}, $$

where we have used (16). Net offshoring flows from the asset-wealthy home country to the asset-poor foreign country when $S_X > 0$, and from foreign to home when $S_X < 0$. Characterizing offshoring patterns using the freeness of trade, we obtain the following:

**Proposition 2** Net offshoring in manufacturing flows from the asset-wealthy country to the asset-poor country for $\varphi \notin (\varphi_{XO}, \varphi_B)$, and from the asset-poor country to the
These stylized labor allocation patterns can be reproduced numerically using $\alpha = 0.95$, $\sigma = 2.5$, $\gamma = 0.5$, $\rho = 0.01$, $\lambda = 0.01$, $l_F = 0.05$, $b = 0.75$ for both panels, and with $\delta = 0.45$ and $\varphi_B = 0.89$ for Panel (a), and $\delta = 0.85$ and $\varphi_B = 0.97$ for Panel (b).

**asset-wealthy country for** $\varphi \in (\varphi_{XO}, \varphi_B)$, where $\varphi_{XO} \in (0, \varphi_X)$.

**Proof:** See Appendix C.

Net offshoring flows for the manufacturing sector are illustrated by the $S_X$ curve in Figure 3, where the vertical axis measures $S_X \in (-b^*, b)$ over the range $\varphi \in (0, 1)$ indicated by the horizontal axis. There are three cases for the direction of net offshoring. Starting from a high level of trade costs over the range $\varphi \in (0, \varphi_{XO})$, the investment income and innovation cost effects dominate the labor income effect ensuring a greater share of industry for home. The market of home is not sufficiently large to support the production of all home-owned firms, however, resulting in net offshoring flows from home to foreign ($S_X > 0$). For a mid-level of trade costs $\varphi \in (\varphi_{XO}, \varphi_B)$, the home market is sufficiently large to attract the production of a larger share of firms, including the production of firms with foreign owners, and net offshoring therefore flows from foreign to home ($S_X < 0$). For a low level of trade costs over the range $\varphi \in (\varphi_B, 1)$, the labor income effect dominates the investment income and innovation cost effects, and the larger market of foreign attracts the greatest share of produc-
tion, with a share of home-owned firms also locating production in foreign, thereby generating net offshoring flows from home to foreign \((S_X > 0)\).

The direction of net offshoring flows in innovation is calculated using the difference between national shares of asset wealth and innovation. For home we have

\[
S_I \equiv b - s_I = \frac{b\omega(1 - \omega^2) - b^*\omega^*(1 - \omega^2)}{(1 - \omega^2)\omega^* + (1 - \omega^2)\omega},
\]  

(25)

where we have used (23). Net offshoring in innovation flows from the asset-wealthy home country to the asset-poor foreign country when \(S_I > 0\), and from foreign to home when \(S_I < 0\).

**Proposition 3** When \(\delta < \delta^*\), net offshoring in innovation flows from the asset-wealthy country to the asset-poor country for \(\varphi \notin (\varphi_{IO}, \varphi_B)\) and from the asset-poor country to the asset-wealthy country for \(\varphi \in (\varphi_{IO}, \varphi_B)\), where \(\delta < \delta^*\) and \(\varphi_{IO} \in (0, \varphi_I)\). When \(\delta > \delta^*\), net offshoring in innovation always flows from the asset-wealthy country to the asset-poor country.

Proof: See Appendix C.

The pattern of innovation offshoring depends on knowledge diffusion. As shown in Figure 3a, where net offshoring flows for innovation are measured on the vertical axis \((S_I \in (-b^*, b))\), when knowledge diffusion is relatively low \((\delta < \delta^*)\), there are three cases. The first case occurs over the range \(\varphi \in (0, \varphi_{IO})\) where home’s greater share of industry provides it with a knowledge spillover advantage that allows it to attract a larger share of innovation. The knowledge spillover advantage is not sufficient, however, to prevent a share of home-owned firms from locating innovation in foreign with the aim of taking advantage of lower high-skilled wages. As a result, net offshoring in innovation flows from home to foreign \((S_I > 0)\). The second case occurs for intermediate trade costs \(\varphi \in (\varphi_{IO}, \varphi_B)\) where home’s share of industry generates a knowledge spillover advantage that is great enough to ensure that all home-owned firms, and a
share of foreign-owned firms, locate innovation in the home country, despite higher high-skilled wages. In this case net offshoring flows from foreign to home \((S_I < 0)\). The third case arises for low trade costs \(\varphi \in (\varphi_B, 1)\) when foreign has a larger share of industry, and all foreign firms and a share of home-owned firms locate innovation in the foreign country to take advantage of greater knowledge spillovers, generating net offshoring flows from home to foreign \((S_I > 0)\).

In general, marginal improvements in knowledge diffusion have ambiguous effects on national labor allocations. We can show, however, that for relatively high values of \(\delta\), we have \(\varphi < \varphi_B, s_I > 0,\) and \(s_X > 0\), with improved knowledge spillovers inducing a fall in the relative wage rate and reductions in the home shares of innovation and production that cause upward shifts in the \(S_X\) and \(S_I\) curves in Figure 3. In particular, as shown in Figure 3b, for sufficiently high knowledge diffusion, the knowledge spillover advantage associated with concentrated industry is relatively weak, creating an incentive for firms to focus on minimizing the cost of employing high-skilled labor when choosing where to locate innovation. As such, process innovation never concentrates fully in one country. Indeed, with perfect knowledge diffusion \((\delta = 1)\), exactly half of all innovation activity is located in each country: \(S_I = b - 1/2 > 0\). Therefore, in the case for \(\delta > \bar{\delta}\), net offshoring in innovation always flows from the asset-wealthy home country to the asset-poor foreign country \((S_I > 0)\).

### 4 Offshoring and Productivity Growth

This section compares the effects of increased industry concentration on net offshoring flows, market entry, and productivity growth. We combine \((9), (11), (12), (15),\) and \((18)\) to obtain the level of market entry and the rate of productivity growth as follows:

\[
\begin{align*}
n &= \frac{(\nu - \rho - \lambda)\omega k}{\omega k l_F - \rho - \lambda}, \\
g &= \frac{(\sigma - 1)\gamma(\omega k l_F - \rho - \lambda)}{\sigma(\nu - \rho - \lambda)} - \rho - \lambda,
\end{align*}
\]
where $\nu = (1 - (\sigma - 1)\gamma)/\sigma \in (0, 1)$ and $\nu > \rho + \lambda$ is necessary for a positive level of market entry, given that we assume $\omega kl_F > \rho + \lambda$ to ensure saddle-path stability. Note that productivity growth is not biased by a scale effect as changes in overall population size ($2Z$) are fully absorbed by adjustments in the number of firms in the market ($N_W$), leaving the level of market entry ($n$) unchanged.$^7$

We use the expressions in (26) to examine how greater economic integration between countries affects market entry and productivity growth. A key feature of the framework is a positive relationship between the unit cost of process innovation ($p_I = 1/(\omega k\theta)$) and the concentration of innovation and production in either home or foreign, as the benefit of improved knowledge spillovers is outweighed by the cost of rising high-skilled wages.$^8$ Returning to (26), an increase in the unit cost of process innovation leads to a greater level of market entry and a faster rate of productivity growth. Specifically, optimal firm-level employment in process innovation falls, resulting in lower overall per-period labor costs ($w_H h_I$) and higher per-period profits ($\Pi = \nu/n - (\rho + \lambda)/(\omega k) - l_F$) that draw new firms into the market ($dn/d(\omega k) < 0$). The decrease in firm-level employment in innovation, however, depresses the rate of productivity growth ($dg/d(\omega k) > 0$). Thus, improved economic integration through a fall in trade costs or a rise in knowledge diffusion affects market entry and productivity growth through adjustments in the unit cost of process innovation.

Beginning with a decrease in trade costs we obtain the following proposition.

**Proposition 4** A reduction in trade costs increases market entry and dampens productivity growth for $\varphi < \varphi_B$, while decreasing market entry and accelerating productivity growth for $\varphi > \varphi_B$.

$^7$There is a large empirical literature concluding that there is no significant relationship between economic growth and population size (Dinopoulos and Thompson 1999; Barro and Sala-i-Martin 2004; and Laincz and Peretto 2006). The framework presented in this paper corrects for scale effects by focusing on the innovation associated with the production technologies of individual product lines, rather than considering R&D at the national level.

$^8$In general the empirical literature reports mixed results for the effect of industry concentration on economic growth. See Gardiner et al. (2011) for a literature survey and for evidence supporting a negative relationship between a number of measures of industry concentration and GDP growth.
Proof: See Appendix D.

A decrease in trade costs affects the unit cost of process innovation indirectly through changes in national shares of production. For \( \varphi < \varphi_B \), lower trade costs increase the concentration of industry in home, raising unit innovation costs. Accordingly, optimal firm-level employment in innovation falls, generating lower per-period labor costs and greater per-period profits that attract new firms into the industry and raises the level of market entry. The decrease in firm-level employment in process innovation, however, depresses the rate of productivity growth. In contrast, for \( \varphi > \varphi_B \), lower trade costs reduce industry concentration in foreign leading to lower unit costs for innovation, a lower level of market entry, and a faster rate of productivity growth.

Next, examining the effects of adjustments in knowledge diffusion we obtain the following result.

**Proposition 5** An improvement in the degree of knowledge diffusion reduces market entry and accelerates productivity growth for \( \varphi < \varphi_B \), but has ambiguous effects on market entry and productivity growth for \( \varphi > \varphi_B \).

Proof: See Appendix D.

An improvement in knowledge diffusion affects the unit cost of process innovation both directly through greater knowledge spillovers that raise labor productivity in innovation and indirectly through adjustments in national shares of production. Thus, on the one hand, as discussed in Section 3, for \( \varphi < \varphi_B \) a sufficient increase in \( \delta \) lower the concentration of industry in home, and both the direct and indirect effects aline to decrease the unit cost of process innovation. The result is greater firm-level employment in innovation, a lower level of market entry, and accelerated productivity growth. On the other hand, for \( \varphi > \varphi_B \), the increase in \( \delta \) has ambiguous effects on national shares of production, with ambiguous results for the unit cost of process innovation, the level of market entry, and the rate of productivity growth.
Focusing on the case for which industry and innovation concentrate in the asset-wealthy country \((\varphi < \varphi_B)\), we now compare the effects of economic integration through lower trade costs on net offshoring flows and productivity growth. Returning to Figure 3, an increase in \(\varphi\) reduces the home shares of innovation and production, thus decreasing \(S_I\) and \(S_X\), and implying that a decrease in net offshoring flows in innovation and manufacturing coincides with a greater level of market entry and a slower rate of productivity growth. In contrast, an improvement in knowledge diffusion shifts the \(S_X\) and \(S_I\) curves upwards in Figure 3. Therefore, increases in net offshoring flows in innovation and manufacturing coincide with a decrease in the level of market entry and a faster rate of productivity growth. With the evidence presented in Baldwin et al. (2001) suggesting that communication costs have fallen at a faster rate than trade costs, our results indicate that an improvement in knowledge diffusion is a plausible explanation for the recent rise in innovation offshoring. In addition, this trend may coincide with faster productivity growth as the unit cost of process innovation falls.

5 Concluding Remarks

This paper has developed a two-country model to examine the relationship between net offshoring patterns in innovation and manufacturing and fully endogenous productivity growth. Central to the model, monopolistically competitive firms invest in process innovation that lowers production costs and drives aggregate productivity growth. The occupational choice of skill-differentiated workers into low-skilled employment in production and high-skilled employment in innovation determines national labor allocations, while the free movement of investment allows manufacturing firms to shift their innovation and production activities freely between countries. These two mechanisms create a tension between accessing the technical knowledge contained in production processes and sourcing low-cost high-skilled labor as firms independently select the optimal locations for innovation and production. A key feature of the model
is a positive relationship between the unit cost of process innovation and the geographic concentration of industry and innovation as the benefit of greater knowledge spillovers is offset by the cost of rising high-skilled wages.

Assuming symmetric labor forces, we characterize location patterns according to trade costs. Specifically, while high trade costs lead to a larger market and the concentration of innovation and production in the asset-wealthy country, when trade costs are low innovation and production concentrate in proximity to the larger market of the asset-poor country. Given these location patterns, we use the level of trade costs to identify three cases for the directions of net offshoring in innovation and manufacturing. For high trade costs, although the asset-wealthy country has greater shares of industry and innovation, the domestic market is not sufficiently large to attract the innovation and production activities of all firms with domestic owners, and net offshoring thus flows towards the asset-poor country. For intermediate trade costs, however, net offshoring flows from the asset-poor country towards the larger market of the asset-wealthy country. Finally, for low trade costs, the net offshoring flows towards the asset-poor country as it maintains greater concentrations of industry and innovation.

Focusing on the case for which the asset-wealthy country has greater shares of industry and innovation activity, we investigate the effects of an improvement in knowledge diffusion between countries, and find that net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country increase as firms offshore innovation to the asset-poor country to take advantage of lower wages for high-skilled workers. The resulting increased dispersion of industry and innovation activity away from the asset-wealthy country results in lower unit costs for process innovation and thus accelerates productivity growth.
Appendix A: Saddle-path Stability

With \( \omega_k = (1 + \delta)\omega^*/(\omega + \omega^*), L + L^* = (\omega + \omega^*)Z, H/\omega + H^*/\omega^* = (\omega + \omega^*)(1 - \omega^*)Z/(2\omega^*)\) and \( I_W = (\omega + \omega^*)(1 + \omega\omega^*)Z/(2\omega^*)\), the share locus (20) and the investment locus (\( n_H = n_L \)) are written in implicit form respectively as follows:

\[
\Omega = \frac{\omega^*}{\omega} - \frac{\delta - \varphi + (1 - \delta \varphi)\omega^* + (1 - \delta)(1 + \varphi)(b - b^*\omega^*)\alpha \rho}{\omega + (1 - \delta \varphi)\omega^* + (1 - \delta)(1 + \varphi)(b^* - b\omega^*)\alpha \rho}
\]

\[
\Phi = \frac{(\sigma - 1)\gamma \omega k}{\sigma(\rho + \lambda)} - (1 - \alpha \rho)(1 - \omega^*)\omega k - 2(1 - \alpha \rho)\omega^* + \frac{\sigma - \alpha}{\alpha \sigma l_F} + \frac{\lambda}{l_F}.
\]

We use these expressions to obtain the following partial derivatives:

\[
\frac{\partial \Omega}{\partial \omega} = -\frac{\omega^*}{\omega^2} + \frac{(1 - \alpha \rho)(\omega - \omega^*)^2\omega^*}{\alpha \rho (b - b^*)(1 + \omega^*)^2\omega^2} > 0,
\]

\[
\frac{\partial \Omega}{\partial \omega^*} = \frac{1}{\omega} + \frac{(1 - \alpha \rho)(\omega - \omega^*)^2}{\alpha \rho (b - b^*)(1 + \omega^*)^2\omega} > 0,
\]

\[
\frac{\partial \Omega}{\partial b} = -\frac{(\delta - \varphi + (1 - \delta \varphi)\omega^* + (1 - \delta)(1 + \varphi)(b^* - b\omega^*)\alpha \rho)}{\omega + (1 - \delta \varphi)\omega^* + (1 - \delta)(1 + \varphi)(b^* - b\omega^*)\alpha \rho} > 0,
\]

\[
\frac{\partial \Omega}{\partial \varphi} = \frac{-\alpha \rho (1 + \delta)^2(b^* - b^*)\omega^2}{(1 + \delta)\omega^* - \omega^2} < 0,
\]

\[
\frac{\partial \Omega}{\partial \delta} = \frac{-\alpha \rho (1 - \delta)^2(1 + \varphi)(b - b^*)(1 + \omega^*)\omega^2}{(1 - \varphi)\omega^* - \omega^2} > 0,
\]

\[
\frac{\partial \Phi}{\partial \omega} = \frac{n \omega^*}{(\omega + \omega^*)\omega} + \frac{2(1 - \alpha \rho)(\omega l_F - \rho - \lambda)\omega^*}{\alpha (\rho + \lambda)(1 + \omega^*)^2 l_F} > 0,
\]

\[
\frac{\partial \Phi}{\partial \omega^*} = \frac{n \omega}{(\omega + \omega^*)\omega^*} + \frac{2(1 - \alpha \rho)(\omega l_F - \rho - \lambda)\omega}{\alpha (\rho + \lambda)(1 + \omega^*)^2 l_F} > 0,
\]

\[
\frac{\partial \Phi}{\partial \delta} = \frac{n}{1 + \delta} > 0.
\]

The slope of the share locus is \((d\omega^*/d\omega)|_{\Omega=0} = -(\partial \Omega/\partial \omega)/(\partial \Omega/\partial \omega^*)\), and is positive or negative depending on the sign of \(\partial \Omega/\partial \omega\). The slope of the investment locus is \((d\omega^*/d\omega)|_{\Phi=0} = -(\partial \Phi/\partial \omega)/(\partial \Phi/\partial \omega^*) < 0\). We study the stability of the steady state described by \( n_H = n_L = n \) and \( \Omega = \Phi = 0 \) using a Taylor expansion of (21) and (22) to obtain the following determinant for the Jacobian matrix \( J_1 \) of the linearized
system:

$$|J_1| = \frac{(1 + \omega \omega^*)(\rho + \lambda)l_F}{2\omega^*} \left( \frac{\partial \Phi}{\partial \omega^*} \right) \left( \frac{d\omega^*}{d\omega} \bigg|_{\Phi=0} - \frac{d\omega^*}{d\omega} \bigg|_{\Omega=0} \right).$$

As the relative wage rate ($\omega$) is a control variable and the level of market entry ($n$) is a state variable, we require one positive and one negative eigenvalue for saddle-path stability. Accordingly, we consider long-run equilibria that satisfy $|J_1| < 0$, which is ensured for $\partial \Phi / \partial \omega > 0$ and $(d\omega^*/d\omega)|_{\Omega=0} > (d\omega^*/d\omega)|_{\Phi=0}$, where $\omega kl_F > \rho + \lambda$ is a sufficient condition for $\partial \Phi / \partial \omega > 0$ and $\partial \Phi / \partial \omega^* > 0$.

**Appendix B: Proposition 1**

The steady-state comparative statics associated with national labor allocations are

$$\frac{d\omega}{db} = -\frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \Omega}{\partial b}, \quad \frac{d\omega^*}{db} = \frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \Omega}{\partial b},$$

$$\frac{d\omega}{d\varphi} = -\frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \varphi}{\partial \varphi}, \quad \frac{d\varphi}{d\varphi} = \frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \varphi}{\partial \varphi},$$

$$\frac{d\omega}{d\delta} = -\frac{1}{|J_2|} \left( \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \delta}{\partial \delta} - \frac{\partial \Phi}{\partial \delta} \frac{\partial \omega^*}{\partial \delta} \right), \quad \frac{d\omega^*}{d\delta} = \frac{1}{|J_2|} \left( \frac{\partial \Phi}{\partial \omega^*} \frac{\partial \delta}{\partial \delta} - \frac{\partial \Phi}{\partial \delta} \frac{\partial \omega^*}{\partial \delta} \right),$$

where $|J_2| = (\partial \Omega / \partial \omega^*)(\partial \Phi / \partial \omega^*)((d\omega^*/d\omega)|_{\Omega=0} - (d\omega^*/d\omega)|_{\Phi=0}) < 0$ for $|J_1| < 0$. These comparative statics are used to derive Proposition 1. First, we define $\varphi_B$ as the threshold value of the freeness of trade for which the denominator of $\partial \Omega / \partial b$ equals zero. Then, we have $\partial \Omega / \partial b < 0$, $\omega^* > \omega$, $s_X > 1/2$, and $s_I > 1/2$ for $\varphi < \varphi_B$, and $\partial \Omega / \partial b > 0$, $\omega^* < \omega$, $s_X < 1/2$, and $s_I < 1/2$ for $\varphi > \varphi_B$.

Next, considering the effects of an increase in $\varphi$, since $\partial \Omega / \partial \varphi < 0$, we have $d\omega/d\varphi < 0$ and $d\omega^*/d\varphi > 0$ for $s_X \in (0, 1)$. As such, when $\varphi < \varphi_B$, $ds_X/d\varphi > 0$ until $s_X = 1$ at $\varphi = \varphi_X$, where $\varphi_X$ is the threshold value of the freeness of trade at which the investment locus and the $\omega^*/\omega = 1/\delta$ line intersect in Figure 2. Similarly, $ds_I/d\varphi > 0$ until $s_I = 1$ at $\varphi = \varphi_I$, where $\varphi_I$ is the threshold value of the freeness of trade at
which the investment locus intersects the $\omega^* = 1$ line. Alternatively, when $\varphi > \varphi_B$, 
$s_X = 0$ until $\varphi \geq \varphi_X^*$, after which $ds_X/d\varphi > 0$, where $\varphi_X^*$ is the threshold value of 
the freeness of trade at which the investment locus and the $\omega^*/\omega = \delta$ line intersect. 
Likewise, $s_I = 0$ until $\varphi \geq \varphi_I^*$, after which $ds_I/d\varphi > 0$, where $\varphi_I^*$ is the threshold level 
of value of the freeness of trade at which the investment locus intersects the $\omega = 1$ line.

The ranking of $\varphi_X$ and $\varphi_I$ depends on the degree of knowledge diffusion. Given 
that $d\omega/d\delta|\varphi=0; \omega^*=0 = -(\partial \Phi/\partial \delta)/(\partial \Phi/\partial \omega) < 0$, an increase in $\delta$ shifts the investment 
locus downwards. Hence, as a rise in $\delta$ rotates the $\omega^*/\omega = 1/\delta$ line clockwise around 
the origin, we can define the threshold value $\delta = \delta$ at which $\varphi_X = \varphi_I$. Thus, we have 
$\varphi_X > \varphi_I$ for $\delta < \delta$ and $\varphi_X \leq \varphi_I$ for $\delta \geq \delta$. It can be similarly shown that $\varphi_I^* > \varphi_X^*$ 
for $\delta < \delta$ and $\varphi_I^* \leq \varphi_X^*$ for $\delta \geq \delta$.

Appendix C: Propositions 2 and 3

First, we find that $b - s_X \geq 0$ for $\omega^*/\omega \leq (b + \delta b^*/(b^* + \delta b)$, where $1/\delta > (b + 
\delta b^*/(b^* + \delta b) > 1$. In addition, evaluating (20) at $\varphi = 0$, we have

$$
\left. \frac{\omega^*}{\omega} \right|_{\Omega=0; \varphi=0} = \frac{\delta + \omega^* + (1 - \delta)(b - b^*\omega^*)\alpha \rho}{\delta + \omega^* + (1 - \delta)(b^* - b^*\omega^*)\alpha \rho} < \frac{b + \delta b^*}{b^* + \delta b},
$$
given that $1 > \alpha \rho$. As $d(\omega*/\omega)|_{\Omega=0}/d\varphi > 0$, there is a threshold value $\varphi = \varphi_{XO} \in (0, \varphi_X)$ at which $b = s_X$. Thus, $b < s_X$ for $\varphi \in (\varphi_{XO}, \varphi_B)$ and $b > s_X$ for $\varphi \notin (\varphi_{XO}, \varphi_B)$. This proves Proposition 2.

Next, we set $b = s_I$ to obtain $b^*(1 - \omega^{*2}) = b^*\omega^*(1 - \omega^2)$. Given $b > b^*$, for 
relative wage combinations that satisfy this condition, we have $\omega^* > \omega$. In addition, 
this condition is strictly concave with a positive slope:

$$
\frac{d\omega^*}{d\omega} = \frac{\omega^{*2}(1 + \omega^{*2})b^*}{\omega^2(1 + \omega^{*2})b} > 0, \quad \frac{d^2\omega^*}{d\omega^2} = -\frac{2b^*\omega^{*2}}{b(1 + \omega^{*2})\omega^3} \left(1 - \frac{(1 - \omega^{*2})(1 + \omega^2)^2}{(1 - \omega^2)(1 + \omega^2)^2}\right) < 0,
$$

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where the second term in parentheses in $d^2\omega^*/d\omega^2$ is less than one for $\omega^*/\omega > 1$. Since the investment locus has a negative slope, it intersects $b_\omega(1-\omega^2) = b^*\omega^*(1-\omega^2)$ once at the threshold value $\varphi = \varphi_{IO} \in (0, \varphi_I)$. Thus, we have $b < s_I$ for $\varphi \in (\varphi_{IO}, \varphi_B)$, and $b > s_I$ for $\varphi \not\in (\varphi_{IO}, \varphi_B)$. The existence of the range $(\varphi_{IO}, \varphi_X)$ depends on the position of the $\omega^*/\omega = 1/\delta$ line in Figure 2. As increases in $\delta$ cause the $\omega^*/\omega = 1/\delta$ line to rotate clockwise around the origin and the investment locus to shift downwards ($d\omega/d\delta|_{\Phi=0} = -(\partial\Phi/\partial\delta)/(\partial\Phi/\partial\omega) < 0$), there is a threshold value $\delta^* = \delta$ at which $\varphi_{IO} = \varphi_X$. Therefore, we have $\varphi_{IO} < \varphi_X$ for $\delta < \delta^*$ and $\varphi_{IO} \geq \varphi_X$ for $\delta \geq \delta^*$, as outlined in Proposition 3.

Finally, as $s_X$ and $s_I$ are increasing functions of $\omega^*/\omega$, we consider the effects of changes in $\varphi$ and $\delta$ on $s_X$ and $s_I$ using

$$
\frac{1}{\Theta_2} \frac{d(\omega^*/\omega)}{d\varphi} = (1 + \omega^*) \left( \frac{n}{\omega\omega^*} + \Theta_1 \right),
$$

$$
\frac{1}{\Theta_2} \frac{d(\omega^*/\omega)}{d\delta} = -\frac{(1 - \varphi)n}{(1 - \delta^2)(1 + \varphi)\omega\omega^*} + \frac{2(1 - \alpha\rho)n}{(1 + \delta)(1 + \omega^*)} - \frac{(1 - \varphi)\Theta_1}{(1 - \delta^2)(1 + \varphi)(\rho + \lambda)},
$$

where $\Theta_1 = 4(1 - \alpha\rho)(\omega k_F - \rho - \lambda)/(\rho + \lambda)(1 + \omega^*)^2a_{d_F} > 0$, and $\Theta_2 = -((\omega^* - \omega)^2\omega^*)/(\alpha\rho(b - b^*)(1 + \omega^*)\omega^3|J_2|) > 0$. Therefore, $d(\omega^*/\omega)/d\varphi > 0$, and we have $dS_X/d\varphi < 0$ and $dS_I/d\varphi < 0$. In addition, for $\varphi < \varphi_B$, we have $d(\omega^*/\omega)/d\delta < 0$, and a sufficient increase in $\delta$ shifts the $S_X$ and $S_I$ curves upwards in Figure 3.

**Appendix D: Propositions 4 and 5**

First, the effects of changes in $\omega k$ on market entry and productivity growth are

$$
\frac{dn}{d(\omega k)} = -\frac{n(\rho + \lambda)}{l_F\omega k - \rho - \lambda},
$$

$$
\frac{dg}{d(\omega k)} = \frac{(\sigma - 1)\gamma l_F}{\sigma(\nu - \rho - \lambda)}.
$$
Then, the following are used with $|J_2| < 0$ to obtain Propositions 4 and 5:

\[
\begin{align*}
\frac{d(\omega k)}{db} &= \frac{2(1 - \alpha \rho)(\omega_k l_F - \rho - \lambda)\omega_k(\omega^* - \omega)^2}{\alpha(\rho + \lambda)(b - b^*)(1 + \omega^* \omega^2)|J|^2} < 0, \\
\frac{d(\omega k)}{d\varphi} &= \frac{2(1 + \delta)(1 - \alpha \rho)(\omega_k l_F - \rho - \lambda)\omega_k(\omega^* - \omega)^3}{\alpha^2(\rho + \lambda)|l_F(1 - \delta)(1 + \varphi)(b - b^*)(\omega + \omega^*)(1 + \omega^* \omega^2)|J|^2}, \\
\frac{d(\omega k)}{d\delta} &= -\frac{2(1 - \alpha \rho)(\omega_k l_F - \rho - \lambda)\omega_k}{\alpha(\rho + \lambda)(1 + \omega^* \omega^2)(\omega + \omega^*)l_F|J|^2} \left( \frac{2\omega^*}{\omega} + \frac{(1 - \delta)^2(1 - \varphi)(\omega^* - \omega)^3}{\alpha \rho (1 + \varphi)(b - b^*)(1 + \omega^* \omega^2)} \right).
\end{align*}
\]

References


