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The container shipping network design problem with empty container repositioning

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ABSTRACT

This paper addresses the design of container liner shipping service networks by explicitly taking into account empty container repositioning. Two key and interrelated issues, those of deploying ships and containers are usually treated separately by most existing studies on shipping network design. In this paper, both issues are considered simultaneously. The problem is formulated as a two-stage problem. A genetic algorithm-based heuristic is developed for the problem. Through a number of numerical experiments that were conducted it was shown that the problem with the consideration of empty container repositioning provides a more insightful solution than the one without.

Keywords: Container transportation; Routing; Empty Container; Heuristic;
Genetic algorithm
1. Introduction

This paper addresses the issue of designing service networks for container liner shipping while explicitly taking into account empty container repositioning. The container shipping industry has been witnessing an overwhelming growth and prosperity in recent years mainly due to China’s economic boom. To cope with the ever increasing container traffic demand, liner companies are increasing their capacity by investing in new containerships.

Freight transport, by any mode, usually generates a significant number of empty vehicle movements caused by the unbalanced directional flows between two specific points. This issue, which is an intrinsic element of vehicle fleet management and overall logistics scheduling process, has received much attention lately due to its significant consequences. If the inventory control fails in locating empty (or available) vehicles in demand points at the requested time, then the following decision has to be made: load rejection or vehicle leasing. Most of the fleet management studies for the land surface modes consider both alternatives as viable.

Likewise, the sea container industry is also confronted with the problem of allocating empty containers. However, in the case of shortage of available containers, load rejection is very unlikely in practice due to the intensive competition in the market. Consequently, an important decision at the operational level is how to transfer empty containers in a timely and efficient manner and/or lease containers. The above context of container shipping allows us to distinguish the entire shipping network design problem which includes decisions on the voyage itinerary, ship size and calling frequency, into two sub-problems: one for the design of networks to serve loaded container traffic and the other for networks to assign empty container traffic to meet cargo demand. In fact, to the authors’ knowledge, all existing studies related to empty container traffic focus only on empty container repositioning.

Essentially, this study proposes an integrated and comprehensive approach which optimizes the whole problem by designing the network with consideration of both full and empty container traffic and this approach is superior to dealing with two separate sub-problems which may lead to sub-optimization. However, it seems that full and empty traffic is dealt with separately for a number of reasons, mainly having to do with the complexity of problem to be solved, non-abandoned container cargo traffic, etc.

Separation of empty container movements from full container movements is appropriate if all cargo demand is satisfied. Contrary to this, if we are able to forgo unprofitable cargo demand whose generated revenue is offset by the associated costs of
empty container relocation, the examination of both full and empty traffic is required in designing underlying service networks because of the interaction between full and empty traffic. The integrated approach of container liner networks should find an optimal structure of service network by selecting a set of ports to be called with profitable traffic and an associated optimal relocation of the empty container fleet to meet the needs of the selected cargo traffic. The objective of the network design is to maximize the company’s profit resulting from the appropriate composition of revenue voyage and the least costly empty container traffic.

In this paper, we propose a design method for containership routing networks that incorporates empty container repositioning among calling ports, which is modeled on a Knapsack problem basis and is reduced to a location routing problem. The proposed problem is solved by a heuristic based on genetic algorithms (GA), in order to find a set of calling ports, an associated port calling sequence, the number of ships by ship size category and the resulting cruising speed to be deployed in the service networks, with the objective of profit maximization for a liner shipping company. An application of the problem to container transportation in Southeast Asia is presented. In the numerical experiments, results are examined by various factors, which may affect routing and the proportion of loaded and empty containers carried on board the ships.

The paper is organized as follows: Section 2 reviews the relevant literature. Section 3 presents the problem formulation. Section 4 demonstrates a solution procedure. Section 5 illustrates computational experiments. Finally section 6 concludes this study and discusses future research directions.

2. Literature review

As there are a number of existing ship routing and related scheduling studies, most of them are covered by the following three major review papers: Ronen (1983, 1993) and Christiansen et al. (2004).

Limited literature exists though for containership routing problems. Lane et al. (1987) try to find the most economical ship size and mix of fleet for a defined trade route with a known trade demand over a finite planning horizon. Claessens (1987) addresses a shipping model of minimizing the costs including opportunity costs such as penalty costs for cargo not shipped due to ship capacity constraints. Rana and Vickson (1988, 1991) discuss the optimal routing for a fleet of containerships operating on a trade route, to maximize the liner shipping company’s profit. Besides the route, the optimal set of calling port sequence is also provided. They assume that non-profitable ports should not be selected as calling
ports on the route. They formulate the problem as a mixed integer non-linear programming model and solve it by using Lagrangean relaxation techniques. Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) use Linear Programming to assign an existing fleet of containerships to a given set of routes based on detailed realistic models of operating costs. Non-linearities stemming from cruising speed and frequency of service on routes are solved before applying the Linear Programming model. Cho and Perakis (1996) present a study to find the optimal fleet size and design of liner routes. The problem is solved by generating a priori a number of candidate routes for the different ships. Then the problem is formulated and solved as a Linear Programming model. Fagerholt (1999) examined the problem of determining the optimal fleet and the corresponding weekly liner routes and he solved it by employing a set partitioning approach as a multi-trip vehicle routing problem. Bendall and Stent (2001) propose a model of determining the optimal fleet configuration and associated fleet deployment plan in a containership hub and spoke application.

Considerable research has also been performed on the subject of empty container management. Gavish (1981) developed a decision support system for vehicle fleet management. Its adaptation to the container fleet problem is straightforward. In his study, prior to making a decision on empty container relocation, owned and leased containers are assigned to the demand points based on the marginal cost criterion. Crainic et al. (1993) treat leased container allocation, which is determined together with empty container relocation. In their study, long term leasing seems to be assumed because the leasing cost is independent from the duration of the lease. Our paper does not deal with long-term leased containers as they can be considered as owned containers. That is, once they are leased, the leasing cost is necessarily applied whether they are used or not. In contrast to Gavish (1981) and Crainic et al. (1993), our study determines simultaneously the allocation of owned and (short-term) leased containers. Cheung and Chen (1998) develop a stochastic model for a sea-borne empty container allocation problem where owned and leased containers are considered to meet the total transportation demand. Their model makes a distinction between long- and short-term leased containers, although it should be pointed out that in their model constant leasing cost is assumed without lease length consideration. Imai and Rivera (2001) deal with fleet size planning for refrigerated containers where they determine the necessary number of containers required to meet predicted future transportation demand. More recently, Choong et al. (2002) developed an integer programming formulation for empty container relocation with use of both long and short-term leased containers. However, the treatment of the short-term leased container in their study is not appropriate, since the cost of the short-term leased containers is independent of the lease length. The studies of
Imai and Rivera (2001) and Choong et al. (2002) deal with empty container distribution in a relatively broad geographical area. In contrast, two recent papers focus on empty container repositioning in the hinterland of a specific port, in spite of the many similarities that exist in theory and in practice with repositioning in a board area. Li et al. (2004) study the empty container allocation in a port with the aim to reduce redundant empty containers. They consider the problem as a non-standard inventory problem with simultaneous positive and negative demand under a general holding cost function. Jula et al. (in press) consider empty container repositioning, which they refer to as empty container reuse, from a different perspective from that of the above studies. Their aim is the reduction of the traffic congestion in the Los Angeles and Long Beach port area caused heavily by empty maritime container traffic. A network flow formulation is constructed, in order to optimize empty container movements from consignees to shippers directly and/or via inland depots. The problem is solved in two phases: the first phase deals with the model transformation to a bipartite transportation network (i.e. a classical Transportation Problem) and the second phase in solving the Transportation Problem by Linear Programming.

Thus, it can be attested that no container ship routing studies and empty container management studies deal with an integrated and simultaneous approach to determine the optimal fleet composition with corresponding routing characteristics and empty container repositioning.

3. Problem description

3.1. Model outline

The containership routing problem addressed in this paper corresponds to the problem of maximizing profit while picking up and delivering liner container cargoes. Liner shipping companies design fixed schedules and routes as a weekly service, which are kept in place for a relatively long time, e.g., for a few months or for a year. Deciding the voyage routing schedule depends upon several factors such as seasonal cargo fluctuations, market requirements, company policy, etc. Therefore, the shipping companies estimate potential cargo demand at each calling port on a weekly basis and try to construct service routes or networks by explicitly taking into consideration incurred costs and corresponding revenues during a specified planning horizon. The decision-making process involves finding the optimal cruising speed (in practice however, the speed is sometimes set in advance and is therefore regarded as an important constraint in finding the optimal routes and calling schedules) and the number of ships for an optimized schedule.

Most container shipping companies assign a number of ships on a particular trade
route, which is characterized by two end ports (i.e., head-end and tail-end ports) and many intermediate calling ports. In order to maximize profit, the companies must decide: ports to be called and the order of calling sequence for the chosen ports.

As the problem must determine the above decision factor, it is the so-called location routing problem. In most existing voyage routes, all ports to be called on the way from the head-end port to the tail-end port (referred to as outbound direction) are not always called on the way back to the head-end port (referred to as inbound direction), as shown in Fig. 1, where ports 1 and 6 are the head- and tail-end ports. In the outbound direction, all intermediate ports except for port 2 are called, whilst only ports 4 and 5 are called in the inbound direction. Note that both outbound and inbound directions have the same calling sequence of ports 4 and 5 in this example. In fact, most existing liner routes do not have such a calling sequence; however, the formation of such a flexible calling sequence may be advantageous for efficient empty container repositioning. To the authors’ knowledge, there is no analysis in the literature that is able to construct such a flexible routing.

The model maximizes profit by forming a single route, which does not necessarily call at all the ports in the trade area. The model assumes a weekly cargo demand for all origin/destination pairs. The other assumptions are as follows:

(a) The demand for empty containers at a port, at a specific point of time, is the difference between the total traffic originating from the port and the total loaded container traffic arriving at the port for that specified time period. This assumption is valid since this study addresses the problem of constructing the optimal shipping network for only one ship operator. However, it should be noted that usually more than one operator is involved in the market; therefore ships operated by different companies call at the specific port. Hence, there are concurrent activities in container traffic pertaining to the port. For example, if a shipping company forgoes specific shipments, then other companies may undertake the rejected shipments. This implies that such a cargo rejection affects the cargo demand of other carriers. However, such an interaction makes the problem very complex; therefore we only focus on the independent decision-making for one company with the supposition that loyalty contracts prevent shippers from
readily shifting to the other carriers.

(b) All the cargo traffic emanating from a port is satisfied if that port is selected on the route.

(c) If a sufficient container quantity is not available at a port, the shortage is fulfilled by leasing containers with the assumption that there are enough containers to be leased.

(d) The total loaded and empty containers transported by a ship must not exceed the ship’s capacity.

3.2. Formulation

The problem of deciding an optimal route (i.e., choosing an optimal set of calling ports and associated calling sequence of ports), can be formulated as a Knapsack problem. The Knapsack problem approach has been widely used not only in ship scheduling problems but also in other general scheduling problems.

The problem consists of two parts. One part is the lower-problem, which identifies the optimal calling sequence of ports for a specific group of calling ports. The other is the upper-problem, which is reduced to the Knapsack problem and chooses the best set of calling ports that associate to the calling sequence found in the lower-problem. The upper-problem [UP] and the lower-problem [LP] may be formulated as follows:

[UP] Maximize \[ \sum_{k \in V} Z^k \rho_k, \]  \tag{1} 
subject to \[ \sum_{k \in V} \rho_k = 1, \]  \tag{2} 
\[ \rho_k \in \{0, 1\}, \forall k \in V, \]  \tag{3} 

where

- \( V \) is the set of groups of calling ports, each of which associates the optimal voyage route that is identified by the [LP]
- \( \rho_k = 1 \) if the route associated with a candidate group of calling ports \( k \) is selected, \( =0 \) otherwise

Given a set of calling ports, [LP] constructs the optimal calling sequence, which associates with it the resulting profit as the objective function value \( Z^k \). For simplicity in formulating [LP], the objective function is denoted as \( Z \). Then, [LP] may be formulated as follows:

[LP] Maximize \[ Z = R - C(y) - P(y), \]  \tag{4} 
subject to \[ \sum_{j \in N} y_{ij} = \sum_{j \in N} y_{ji}, \forall i \in N, \]  \tag{5}
\[ \sum_{i \in S} \sum_{j \in S} y_{ij} \geq 1 \quad \forall S \subseteq N, \quad (6) \]

\[ y = \{ y_{ij} | i \in 1,\ldots,N; j \in 1,\ldots,N \}, \quad (7) \]

\[ y_{ij} \in \{0,1\} \quad \forall i, j \in N, \quad (8) \]

where,

- \( N \) set of calling ports for \( k \in V \)
- \( S \) non-empty subset of \( N \)
- \( C(\cdot) \) shipping cost function of selected arcs \( (i, j) \)
- \( P(\cdot) \) empty container related (or penalty) cost function of selected arcs \( (i, j) \)
- \( R \) revenue associated with \( k \in V \)
- \( y_{ij} = 1 \) if a ship sails on arc \( (i, j) \), \( = 0 \) otherwise

The decision variables are \( y_{ij}s \). The objective function (4) is the maximization of the total profit. Constraint set (5) ensures that a calling ship at a port must depart from that port. As shown in Fig. 1 where it is envisaged that \( N \) contains all ports 1-6 except for 2, the route must connect all the ports in \( N \). Constraints (6), therefore, guarantee that all the ports are connected each other through the formed route. The constraints, in other words, guarantee that there are one or more directed arcs in total between any nodes in any subset of ports, \( S \), and those not in \( S \), resulting in that there is no such a sub-tour that does not visit all the nodes in \( N \). Eq. (7) defines a vector \( Y \) to be comprised of \( y_{ij}s \) for a formed voyage route.

Given a freight rate for the origin-destination port pair, the revenue generated is defined by the set of calling ports with the assumption that the published rate is applied independent of the cargo traffic itinerary which is based on the resulting voyage route. \( R \) is defined as follows:

\[ R = \sum_{i \in N} \sum_{j \in N} F_{ij} x_{ij}, \quad (9) \]

where \( F_{ij} \) is the freight rate of cargo from ports \( i \) to \( j \) and \( x_{ij} \) is cargo traffic from ports \( i \) to \( j \).

Shipping costs depend on a variety of ship factors and transportation demand on the route. In this paper, therefore, the shipping cost is expressed as the sum of the costs regarding arcs on candidate routes. In the following subsections, we provide relevant cost functions. See Imai (1989) for details.
3.3. Shipping cost function

Shipping costs are made up of two components: operating and capital costs. In general, the capital cost includes the cost regarding the ship itself, while the operating cost includes the costs of fuel, lubricant and port entry. These costs are defined as below:

\[ C = CS + CP, \quad (10) \]
\[ CS = C^C + C^F, \quad (11) \]
\[ CP = C^E + C^H, \quad (12) \]

where,

- \( CS \) ship related costs
- \( CP \) port related costs
- \( C^C \) ship’s other costs, which are not incurred in proportion to the cruise distance
\[ (C^C = C^M + C^D + C^R + C^I + C^P) \]
- \( C^D \) ship’s depreciation cost
- \( C^E \) port entry cost
- \( C^F \) fuel and its related cost
- \( C^H \) cargo handling cost
- \( C^I \) insurance cost
- \( C^M \) crew cost
- \( C^R \) interest
- \( C^P \) repair and maintenance cost

The detailed cost functions are illustrated below.

(1) Ship related costs

The market report (Drewry, 2001) investigates ship related costs (actually the time-charter cost) by various ship sizes. As a result of a regression analysis that was performed based on the above cost data, we obtained the following linear cost ($US per day) model using the TEU (twenty-foot equivalent unit) capacity as the independent variable:

\[ C^{CD} = 6.54 \times \text{TEU} + 1422.52. \quad (13) \]

This regression model provides a good prediction, since its coefficient of determination \( R^2 \) is 0.9982.

Given the number of voyages offered yearly, which is easily calculated as it is based on round trip duration, the total cost is the product of the cost per voyage and the cost of a deployed ship. This enables us to concentrate only on the evaluation of the voyage cost
The ship’s other costs, $C^C$, are computed from the multiplication of the ship’s other daily costs, $C^{CD}$, and the time duration of the voyage, $\left(\frac{DIST}{24 \cdot v} + IDLE\right)$:

$$C^C = C^{CD}\left(\frac{DIST}{24 \cdot v} + IDLE\right), \quad (14)$$

$$IDLE = \sum_{i\in N} \left(\sum_{j\in N} \left(e_i + e_j\right)\left(v_{ji} + x_{ji}\right) + f_i + f'_i\right) / 24, \quad (15)$$

where,

$DIST$  round trip distance (nautical miles)  

$Dist_{ij}$  cruising distance from ports $i$ to $j$ (nautical miles)

$IDLE$  stay time at port (days), which is associated with a given group of calling ports

$e_i$  handling time (loading or unloading) per container at port $i$

$f_i, f'_i$  standby times for departure and arrival at port $i$

$w_{ji}$  the number of empty containers carried from ports $j$ to $i$

$v$  cruising speed (knots)

The fuel cost $C^F$ of a voyage, which in this case includes also lubricant cost, is defined by the following equation:

$$C^F = \left(\frac{C_{Fuel}^F R_{Fuel}^F + C_{Lab}^F R_{Lab}^F}{A}\right) \cdot DS^2 \cdot v^2 \cdot DIST, \quad (16)$$

where,

$C_{Fuel}^F$  fuel cost ($US/metric ton$)

$R_{Fuel}^F$  fuel consumption (g/hp/h)

$C_{Lab}^F$  lubricant cost ($US/metric ton$)

$R_{Lab}^F$  lubricant consumption (g/hp/h)

$DS$  displacement

$A$  admiralty coefficient (a parameter used for the naval architecture. See Tupper, (1996))

The derivation of $C^F$ can be found in the Appendix.

The overall ship related cost $CS$ is:

$$CS = C^{CD}\left(\frac{DIST}{24 \cdot v} + IDLE\right) + \left(\frac{C_{Fuel}^F R_{Fuel}^F + C_{Lab}^F R_{Lab}^F}{DS^3 \cdot v^2 \cdot DIST}\right), \quad (17)$$

If we take a partial derivative of Eq. (17) by cruising speed $v$ and set the resulting
equation as zero as follows, then the optimal cruising speed \( v^* \) is defined by Eq. (18).

\[
\frac{\partial CS}{\partial v} = -\frac{C^{CD} \cdot DIST}{24 \cdot v^2} + \frac{2 \left( C^{Fuel} R^{Fuel} + C^{Lab} R^{Lab} \right) \cdot DS^3 \cdot v \cdot DIST}{A} = 0,
\]

\[-C^{CD} \cdot A + 48 \left( C^{Fuel} R^{Fuel} + C^{Lab} R^{Lab} \right) \cdot DS^3 \cdot v^3 = 0,
\]

\[
v^* = \left\{ \frac{C^{CD} \cdot A}{48 \left( C^{Fuel} R^{Fuel} + C^{Lab} R^{Lab} \right) \cdot DS^3} \right\}^{\frac{1}{3}}\quad (18).
\]

The cost at the optimal speed and those at different speeds around the optimal one are plotted in Fig. 2.

\[
\text{Fig. 2}
\]

(2) Port related costs

The port entry cost is given by Eq. (19).

\[
C^E = \sum_{i \in N} \sum_{j \in N} Q_{ij} v_{ij},\quad (19)
\]

where \( Q_{ij} \) is the entry cost per call at port \( i \). Note that a selected arc associates two calling ports \( i \) and \( j \); however only one of them should be counted in the entire voyage.

The handling cost \( C^H \) is associated with a given group of calling ports for [LP] since we assume that a ship undertakes all the cargo demand emanating from calling ports. Since \( C^H \) is the sum of handling costs for all calling ports which are incurred by loaded and empty containers handled at these ports, it is defined by:

\[
C^H = \sum_{i \in N} H_i \sum_{j \in N} (x_{ij} + x_{ji} + w_{ij} + w_{ji}),\quad (20)
\]

where \( H_i \) is the handling cost per container (TEU) at port \( i \).

3.4. Penalty cost function

We consider an optimal route configuration taking into consideration empty container repositioning among calling ports. Liner shipping companies are generally faced with an enormous level of imbalanced cargo traffic between trade sections. This imbalance
creates some costs (referred to as *penalty costs*) as a number of unproductive tasks have to be performed such as the reposition of empty containers from excessive points to demand points, storage of empty containers in place for future demand and leasing containers to meet urgent cargo demand. For the task of container repositioning we assume that no costs are associated with it, because it is performed using the excess capacity on their own ships.

The penalty cost function, $P$, is given by the following equation where a virtual calling sequence shown in Fig. 3 is assumed (the details of the virtual calling sequence are described later):

$$P = \sum_{i \in NV} \left( a_i ST_i + b_i LS_i \right),$$  \hspace{1cm} (21)

$$E_i = \max\{P_i - D_i, 0\} \quad \forall i \in NV ,$$  \hspace{1cm} (22)

$$S_i = \max\{D_i - P_i, 0\} \quad \forall i \in NV ,$$  \hspace{1cm} (23)

$$LS_i = S_i - \sum_{j(x) \in NV} w_{ji} \quad \forall i \in NV ,$$  \hspace{1cm} (24)

$$ST_i = E_i - \sum_{j(x) \in NV} w_{ji} \quad \forall i \in NV ,$$  \hspace{1cm} (25)

$$\sum_{p \in M^i q \in M^i (p < q)} (x_{pq} + w_{pq}) \leq CAP \quad \forall i \in N ,$$  \hspace{1cm} (26)

where,

- $a_i$ storage cost at port $i$ (SUS/TEU)
- $b_i$ short-term leasing cost at port $i$ (SUS /TEU)
- $LS_i$ the number of lease containers at port $i$ (TEU)
- $ST_i$ the number of containers stored at port $i$ (TEU)
- $NV$ the set of nodes in the virtual calling sequence
- $D_i$ cargo traffic departing from port $i$ (TEU)
- $P_i$ cargo traffic destined for port $i$ (TEU)
- $E_i$ the number of excess containers at port $i$ (TEU)
- $S_i$ the number of demanded containers at port $i$ (TEU)
- $M^i$ the set of ports in the virtual calling sequence with port $i$ in the original sequence being port 1
- $CAP$ ship capacity (TEU)

Eq. (22) defines excess containers. If $P_i - D_i$ is negative, i.e., no excess containers exist, $E_i$ is set to zero, otherwise $E_i$ is set $P_i - D_i$. Eq. (23) defines shortage of empty containers. Eq. (24) defines the number of containers to be leased, while Eq. (25) specifies the number of containers to be stored at a port. Inequality (26) guarantees that
empty containers are transported in an excess space of a ship. Fig. 3(b) shows the virtual calling sequence, which was converted from the original calling sequence of Fig. 3(a). The virtual calling sequence includes virtual nodes that represent nodes to be visited more than once in the original calling route. The set of nodes in the virtual sequence is denoted by \( NV \). In (26), \( M^i \) is defined as the set of ports in the virtual sequence where the voyage starts from node \( i \) in the original sequence. Fig. 3 shows \( M^4 \) in the example of five ports in the original sequence. \( M^4 \) has the nodes including virtual ones in sequence for node 4 in Fig 3(a) corresponding to node 1 in Fig. 3(b). In the virtual sequence, traffic, regardless of loaded or empty, from an origin port to another port with a smaller port number than the origin port passes through port 1 in the virtual sequence. Note that in Fig. 3 when \( i = 4 \), there are two orientations for the round trip voyage: one is itinerary \((4, 5, 6, 9, 5, 4, 1)\) and the other is \((4, 1, 4, 5, 6, 9, 5)\), for both of which the virtual sequence is \((1, 2, 3, 4, 5, 6, 7)\); therefore, in such a case we must examine constraint (26) for \( i \) in both orientations.

Assumption (c) in Section 3.1 considers that there is always enough quantity of leased containers; however, this may not always be the case in reality. In such a case and for the purpose of applying the model in practice, if a specific port does not have enough quantity of containers for lease, we may set a huge value for \( b_i \). This prevents us from generating a solution with leasing a number of containers due to the extremely high penalty cost.

4. Solution procedure

This section describes a solution procedure for this problem, which is categorized as a combinatorial optimization problem. As defined the upper-problem of the ship routing problem, \([UP]\) is the Knapsack problem, which is NP-complete (Papadimitriou and Steiglitz, 1982); therefore the ship routing we are concerned is also NP-complete. This implies that there is no efficient exact algorithm for this problem. From this point of view, this paper proposes a heuristic to nearly optimize the solution by employing genetic algorithm (GA).

The GA algorithm we implement solves \([UP]\) and \([LP]\) simultaneously. The GA generates better solutions by crossover and mutation at each iteration (or generation) of an entire solution process. Generating offsprings of parents, which have been generated up to
the previous iteration, corresponds to solving [LP] and choosing better one among the candidate solutions is equivalent to solving [UP].

We need to design the “genetic representation” (or chromosome) of the candidate solution, namely, the coding of a combination of calling ports and associated calling sequence. In general, there are two kinds of genetic operations: crossover and mutation. Crossover generates offspring by combining both parents to create better individuals. Mutation produces spontaneous random change in various chromosomes. In this paper, we modify the representation of chromosome and the operations of crossover and mutation, as they are provided by Inagaki et al. (1999), to conform them to our problem.

4.1. Genetic representation

Fig. 4 illustrates the corresponding formation of Fig. 1. The length of the string of digit is the number of candidate calling ports, without consideration to them being selected or not, from one port to another including intermediate ports on the way to the latter and those on the way back to the former. Note that similar to calling formation described in section 3.3, a specific port is assigned a different number whenever is called in the entire voyage. In Fig. 4, the ports, which are not all called necessarily, on the way back from ports 6 to 1 of Fig. 1 have the numbers 7 through 11 associated with them. Therefore, for example, port 5 in Fig. 1 has two different numbers 5 and 7 in Fig. 4. An arc from ports \( m \) to \( n \) as shown in Fig. 1 is presented as \( n \) is housed in \( m \)th locus. Such linkages between two calling ports are chained to each other to form an entire voyage. Loci, which are equivalent to uncalled ports, house any port numbers arbitrarily.

4.2. Fitness and selection

A selection criterion is used for choosing two parents to apply the crossover operator. A fitness value reflects the goodness of an individual, compared with the other individuals in the populations. In this study, a fitness value corresponds to a value of the objective function, namely, the company’s profit. We adopt two selection methods: an elitist-preserve strategy and a roulette-wheel selection. The former method is used when two higher rank individuals are unconditionally preserved to the next generation and the latter is employed to randomly pick up a superior individual from the remainder.
4.3. Crossover

The crossover scheme contributes powerfully to the success of the GA. The crossover scheme should be capable of reproducing a new feasible solution (or offspring) by combining good characteristics of both parents. A generated offspring should present a round trip on a complete route. In order to keep the feasibility, the crossover operation is performed in the following manner (Fig. 5 shows an example with a new offspring created by crossover and presents the resulting routes): First, define port 1 as the starting port of a voyage. Focusing on digits in the locus corresponding to the origin port in the two selected parents, select a digit arbitrarily from them and store the selected digit in that locus of offspring. Second, focusing on digits, from the parents, located in a locus, which is defined by the digit just housed in offspring; choose either digit randomly and store it in the same locus of that offspring. Repeat this procedure until a round trip is completely formed. Finally in loci equivalent to unvisited ports, digits from either of the parents in the corresponding loci are stored. Furthermore, if an infeasible chromosome as shown in Fig. 6 is generated, perform crossover procedure from the beginning over again.

Figs. 5 & 6

4.4. Mutation

Mutation introduces random changes to the chromosome, and keeps the diversity of individuals. Fig. 7 presents an example of processing of the mutation operator and associated routes. Mutation randomly chooses a locus and houses a digit in that locus, that is chosen randomly from any of the ports in the service area. If an infeasible chromosome is generated, operate mutation again.

Fig. 7

5. Computational experiments

This section presents an application of the problem to container transportation in Southeast Asia. We considered a number of impact factors to the formation of a shipping
route.

The algorithm is coded in Fortran 77 and is run on a DELL Dimension 8250 with 2.40 GHz Pentium IV processor. In order to assess the solution quality of the GA, we compared approximate solutions by the GA with the optimal solutions of the same problems being solved by the Brute force method. Due to the computational limitation of these methods, we tested small illustrative cases of the problem with 5-8 ports in the trade area. The result was that the GA found the optimal solution for every problem.

Based on preliminary experiments with the above small problem cases, parameters of the GA were set as follows: population size=300, maximum number of generations=200, crossover rate=0.9 and mutation rate=0.08.

5.1. Parameter settings for the experiments

Settings of parameters for the experiments are as follows:

1. Potential calling ports (20 ports): Tokyo, Yokohama, Shimizu, Nagoya, Osaka, Kobe, Moji, Hakata, Busan, Shanghai, Keelung, Kaohsiung, Hong Kong, Ho Chi Minh, Manila, Leam Chabang, Bangkok, Port Klang, Jakarta and Singapore.

2. The time horizon: 52 weeks.

3. The calling frequency per year: 52.

4. The turnaround time of a ship: less than or equal to 21 days.

5. Ship sizes: 500, 1000, 1500, and 2000 TEUs.

6. Handling time at each port \((e_j): 0.042 \text{h/TEU.}\)

7. Standby time for departure or arrival at each port \((f_i, f_i'): 2 \text{h each.}\)

8. The handling cost at each port: $US200/TEU.

9. The storage cost at each port \((a_j): $US300/TEU.

10. The short-term leasing cost at each port \((b_j): $US300/TEU.

11. Fuel and lubricant costs \((C_{Fuel}, C_{Lab}): $US170/metric ton and $US1000/metric ton, respectively.

12. Fuel and lubricant consumptions \((R_{Fuel}, R_{Lab}): 140g/hp/h and 4g/hp/h, respectively.

13. The cost per entry at each port \((Q_i): =1.95 \times CAP + 5200.0 \text{(US per entry).}\)

14. Displacement \((DS): = 26.96 \times CAP + 3453.36 \text{(tons).}\)

15. Admiralty coefficients \((A) \text{ for each ship size (500, 1000, 1500 and 2000TEU): 250, 300, 350 and 400, respectively.}\)

For parameters (6)-(13) data was obtained from surveys that were conducted with shipping and stevedoring companies as well as with port authorities in Japan. It is understood of course that the port related costs such as handling, storage, and port entry
costs, vary from port to port. However, due to the lack of detailed data we assume that they are the same for all the ports under consideration. If the actual costs were reflected in the following case studies, the solutions would have been more insightful.

5.2. Case studies

Throughout the experiments the sensitivity of some factors was examined to determine their influence on the solution. The first factor is the impact of the penalty cost coefficients \( (a_i, b_i) \) (i.e., storage and leasing cost). Companies pay considerable attention to the costs related to empty containers nowadays. It is likely that each company assigns a different value to the repositioning cost. Thus, we consider three levels for the empty container-related costs such as: basic cost, twice as much as the basic cost and four times as much.

The second factor is the impact of taking empty containers into consideration. In other words, we look into the difference in the gained profit by the two proposed solutions: the one identified by the problem we propose (case 1) and the other by the problem without consideration of empty container movement (case 2). Calculations for case 2 were also performed by GA, but the problem employed for them did not take into account empty container distribution (i.e., variables \( w_{ij} \)'s are not included in the formulations) and the associated empty container-related costs (or penalty costs) in the objective function. Note that the problem without empty distribution is hereafter referred to as the based problem.

After the based problem was solved, the necessary empty container traffic was distributed in the shipping network, whilst relevant constraints were satisfied and relevant penalty costs were added to the profit of the resulting objective function value. The profit resulting from the above process is the one for case 2.

It seems that in order to keep the sailing schedule, ships must increase cruising speed if the handling time increases, since more empty containers are handled due to the resulting inefficient empty traffic. At the same time, increasing movement of empty containers may also raise the operating costs for the same reason.

5.3. Experimental results

Port-to-port traffic of loaded containers per week is estimated by using several data sources such as the United Nations (1998) and the official web page of each port, etc. Table 1 shows the weekly throughput of import and export containers at each port based on the estimated traffic.
We first look into the best ship size in TEUs. We computed a specific problem sample by GA 50 times by varying the initial arrangement of genes. Fig. 8 portrays the convergence of profit in average of 50 runs during genetic iterations by four different ship sizes. For each ship size, we assumed three different levels of penalty cost (indicated by x1, x2 and x4). While there are no significant differences in profit by the different penalty cost levels, as expected the highest profit is achieved with the deployment of the least cost ship. The most profitable ship size is 1000TEUs.

Table 2 illustrates comparisons between cases 1 and 2 by the most profitable ship size, 1000TEUs, showing the 5 best solutions for each case. The best solutions in case 1 are centered on the number of deployed ships=3, whilst the figures may be fractional. As mentioned before, case 2 solutions are calculated by adding empty container distribution to the solutions of the based problem. Note that in case 2, the based problem does not take into account empty container distribution and therefore transports more loaded containers. As expected, case 1 results in being more profitable than case 2, as it consists of less revenue but also of much less shipping and penalty costs. Interestingly, case 2 has a complicated and inefficient empty container distribution. As the based problem has not considered empty traffic, a lot of loaded containers are transported due to the fact that no ship space is reserved for empty container transfer in the solution of the based problem and consequently a huge shortage of empty containers is observed. This shortage is covered through expensive leasing. Table 3 shows that, as loaded traffic increases, the turnaround times at ports become longer and consequently a higher cruising speed is needed to maintain the service with three ships.
Lastly we provide some insight about the computation time. Referring to Fig. 8, the best solution within those generated by the GA is found up to the 50th generation though the profit is an average value over 50 calculations with a different initial solution (or gene). Table 4 illustrates the profit and CPU time at a different generation in solving a problem with a ship capacity of 1000TEUs, whose solution corresponds to solution #1 for case 1 in Table 2. From the viewpoint of computational efficiency, Fig. 8 and Table 4 suggest that the computation should terminate at most at the 100th generation.

Table 4

In general as the problem size increases the computation time increases as well. This is the case for this problem. In the chromosome manipulation process, the computation time does not significantly increase in spite of increase in the size of chromosome representation with more calling ports involved in the problem. However, as mentioned before the solution procedure repeats genetic operations such as crossover and mutation till those generate chromosomes, which correspond to feasible solutions to the problem. Normally such genetic operations more unlikely generate feasible chromosomes for big problems (i.e., problems with more calling ports) than for small ones. Consequently, the overall computation time increases with larger problem instances.

6. Conclusions

This study addressed the problem of container liner shipping network construction by explicitly taking into account empty container distribution. Whilst there is huge literature on ship routing and scheduling problems, few studies treat the design of container shipping network and none of them incorporate the problem of repositioning and leasing of empty containers. In this paper, this problem was dealt with by forming a shipping network with the assumption that necessary empty container repositioning is performed using spare space on ships operated and containers are leased when empty containers do not arrive at the demand points in time. GA is employed for implementing a solution method for the problem.

Based on the computational experiments that we conducted, the following conclusions can be reached: Due to the empty container flow that was treated in this problem, the handling time and associated costs at ports are smaller than those by using the
Based problem. As a result, the problem with empty distribution results in being able to cruise at a slower speed due to the efficient empty container distribution and thus save considerably the fuel costs.

The proposed approach is very useful in assessing potential shipping networks from both strategic and tactical viewpoints, since the design of the container shipping network without consideration of the empty container traffic eventually becomes very costly due to less efficient empty container distribution associated with the resulting network.

In practice, there is a fierce competition among shipping companies; therefore load rejection, which was considered in this study, may be unlikely even in the case of a fully utilized ship capacity. A mitigation of this restriction may be an interesting topic for future research.

Appendix. The derivation of the fuel cost, \( C^F \)

The fuel cost, \( C^F \), is defined as Eqs. (A.1) and (A.2):
\[
C^F = R_{\text{Fuel}} \cdot HP \cdot C^F \cdot \text{HOUR},
\]
where \( \text{HOUR} \) is the time duration of a round trip voyage and \( HP \) is the engine horsepower, which is defined as follows (Tupper, 1996):
\[
HP = DS^3 \cdot v^3 / A.
\]

The lubricant cost, \( C^L \), can be given by (A.3):
\[
C^L = R_{\text{Lab}} \cdot HP \cdot C^L \cdot \text{HOUR}.
\]

As \( C^L \) is proportional to the voyage time like \( C^F \), \( C^L \) can be included in \( C^F \); then the fuel cost with the lubricant cost is now expressed as:
\[
C^F = \frac{\left( R_{\text{Fuel}} + R_{\text{Lab}} \right) \cdot DS^3 \cdot v^3 \cdot \text{HOUR}}{A}.
\]

Since \( \text{HOUR} \) is defined as \( \text{HOUR} = \frac{\text{DIST}}{v} \), \( C^F \) can be rewritten as Eq. (A.5) or (16).
\[
C^F = \frac{\left( R_{\text{Fuel}} + R_{\text{Lab}} \right) \cdot DS^3 \cdot v^2 \cdot \text{DIST}}{A}.
\]
References
Fig. 1. Example of ship’s itinerary
Fig. 2. Ship related costs at various speeds
\[ N = \{1, 4, 5, 6, 9\} \]

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<thead>
<tr>
<th>Ports in calling sequence</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>5</th>
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<th>1</th>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

\[ M^4 = \{1, 2, 3, 4, 5, 6, 7\} \]

Fig. 3. Calling sequence transformation for \( M^4 \)
Fig. 4. Chromosome representation
<table>
<thead>
<tr>
<th>Locus</th>
<th>1</th>
<th>2</th>
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<th>5</th>
<th>6</th>
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<th>10</th>
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<td>4</td>
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<td>6</td>
<td>8</td>
<td>11</td>
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<td>9</td>
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<tr>
<td>Parent2:</td>
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<td>6</td>
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<td>9</td>
<td>9</td>
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<td>Offspring1:</td>
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<td>2</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>7</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

P1: 1 → 3 → 4 → 5 → 6 → 4 → 5 → 1
P2: 1 → 4 → 6 → 3 → 4 → 1
O1: 1 → 4 → 6 → 4 → 5 → 1

Fig. 5. Example of crossover processing
Fig. 6. Infeasible chromosome having lethal genes
Fig. 7. Example of mutation processing

<table>
<thead>
<tr>
<th>Locus</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>6</td>
<td>8</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>11</td>
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</tbody>
</table>

O1: 1 → 4 → 6 → 4 → 5 → 1
O1': 1 → 4 → 6 → 4 → 5 → 2 → 1
Table 1. Weekly throughput at ports (TEUs)

<table>
<thead>
<tr>
<th>Port</th>
<th>Import</th>
<th>Export</th>
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</thead>
<tbody>
<tr>
<td>Tokyo</td>
<td>879</td>
<td>1076</td>
</tr>
<tr>
<td>Yokohama</td>
<td>906</td>
<td>1181</td>
</tr>
<tr>
<td>Shimizu</td>
<td>196</td>
<td>271</td>
</tr>
<tr>
<td>Nagoya</td>
<td>382</td>
<td>790</td>
</tr>
<tr>
<td>Osaka</td>
<td>282</td>
<td>506</td>
</tr>
<tr>
<td>Kobe</td>
<td>1008</td>
<td>1533</td>
</tr>
<tr>
<td>Moji</td>
<td>196</td>
<td>260</td>
</tr>
<tr>
<td>Hakata</td>
<td>288</td>
<td>382</td>
</tr>
<tr>
<td>Busan</td>
<td>1925</td>
<td>1467</td>
</tr>
<tr>
<td>Shanghai</td>
<td>779</td>
<td>914</td>
</tr>
<tr>
<td>Keelung</td>
<td>325</td>
<td>417</td>
</tr>
<tr>
<td>Kaohsiung</td>
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<td>539</td>
</tr>
<tr>
<td>Hong Kong</td>
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<td>392</td>
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<tr>
<td>Ho Chi Minh</td>
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</tr>
<tr>
<td>Manila</td>
<td>382</td>
<td>624</td>
</tr>
<tr>
<td>Leam Chabang</td>
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<td>69</td>
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<td>Bangkok</td>
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<td>Singapore</td>
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<td>912</td>
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Fig. 8. Convergence of GA solution by different ship sizes
Table 2. Comparison between cases 1 and 2 by 1000TEU ship

<table>
<thead>
<tr>
<th>Case</th>
<th>Sol.</th>
<th>Calling sequence</th>
<th>Profit (x$US10^4)</th>
<th>Revenue (x$US10^4)</th>
<th>Shipping cost (x$US10^4)</th>
<th>Penalty (x$US10^4)</th>
<th>Cruising speed (knots)</th>
<th>Fleet size</th>
<th>Distance (nautical miles)</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1-2-17-20-15-11-2-1</td>
<td>4120.3</td>
<td>10394.0</td>
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<td>6385</td>
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1: Tokyo, 2: Yokohama, 3: Shimizu, 4: Nagoya, 5: Osaka, 6: Kobe, 7: Moji, 8: Hakata, 9: Busan, 10: Shanghai
11: Keelung, 12: Kaohsiung, 13: Hong Kong, 14: Ho Chi Minh, 15: Manila, 16: Leam Chabang, 17: Bangkok
18: Port Klang, 19: Jakarta, 20: Singapore
Table 3. The mean handling time for Cases 1 and 2 (1000 TEUs)

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<tr>
<th>Case #</th>
<th>Sol.#</th>
<th>Mean handling time (h)</th>
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<td></td>
<td></td>
<td></td>
<td>Loaded container</td>
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<td>1</td>
<td>1</td>
<td>143.1</td>
<td>10.3</td>
<td></td>
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<td>2</td>
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Table 4. Profits and CPU times by generation

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<td>4120.3</td>
<td>4120.3</td>
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<td>CPU time (sec)</td>
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<td>41.9</td>
<td>63.2</td>
<td>85.1</td>
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