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Sequential Entry in a Vertically Differentiated Market*

Masahiro ASHIYA
Faculty of Economics
Nagoya City University

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Abstract

This paper considers a sequential entry game of homogeneous firms in a vertically differentiated market. A firm can choose any variety of products, with a fixed cost per product. Each product can be withdrawn afterwards without exit costs. Then each firm chooses one product at most in equilibrium because of a commitment problem. The first firm chooses the highest quality if the fixed cost is so large that subsequent entry is blockaded. It chooses middle quality to deter entry of a low-quality firm if the fixed cost decreases. Hence everyone becomes worse off as the entrant becomes more dangerous.

Keywords: Entry deterrence; Vertical differentiation; Brand proliferation; Commitment.
JEL Classification Codes: D43, L13.

Address: Yamanohata, Mizuho-ku, Nagoya, 467-8501 Japan,
Email: ashiya@econ.nagoya-cu.ac.jp

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1. Introduction

In a regional gasoline market, a retailer can build any kind of gas station: a full-service station, a self-service station, or both. Let us consider the optimal strategy of the retailer. Choosing both types of stations and screening its customers is the best policy without competition. It is not the best, however, if entry occurs subsequently. The reason, which Judd (1985) first points out, is that it faces a commitment problem. Suppose that the incumbent retailer builds both a full-service stand and a self-service stand, and that a newcomer builds a self-service stand. Then the profit the incumbent can earn from its self-service stand is very small because competition drives down the self-service prices of both firms. Furthermore, this competition produces a negative effect on the profit from the full-service stand. Consequently, on condition that the exit cost is relatively small, it is better for the incumbent retailer to withdraw the self-service stand and relax competition.

This paper formalizes the above idea and considers a sequential entry game of homogeneous firms in a vertically differentiated market. A firm can choose any variety of products, with a fixed cost per product. Each product can be withdrawn afterwards without exit costs. Then the entry threat of the subsequent firms forces the first entrant not to proliferate its brand. The intuition is as follows. Suppose that the first entrant fills up the market by some kinds of products and that the second firm enters near to one of them. Then the first entrant withdraws competing products in equilibrium, since withdrawal of the products near to the second firm relaxes competition and increases profit from the remaining products. What makes the matter worse is that the second firm anticipates this incentive of the first firm. It enters the most profitable position in the market, and kicks out the products of the first entrant. This argument indicates that choosing two or more kinds of products facilitates entry: it warrants the second firm a larger and a better niche.

This paper proves rigorously that each firm chooses one kind of product at most in equilibrium. Moreover, it investigates the transition of the equilibrium outcome by gradually decreasing the fixed cost. The transition consists of five phases: one firm enters the market in Phase 1 and 2, and two firms enter the market in Phase 3, 4, and 5. 1
In Phase 1, when the fixed cost is large enough to blockade entry of the second firm, the first entrant selects the good of the highest quality. In Phase 2, when choosing top quality cannot blockade new entry, it chooses middle quality to prevent entry in a low-quality market. In Phase 3, when it cannot deter entry of the second firm, the first entrant chooses the highest quality to secure its profit. The second firm chooses a product of low quality to mitigate competition. When the fixed cost decreases still further, the first and the second entrant must choose similar products to deter entry of the third firm between them. In Phase 4, when the fixed cost is relatively large, the first firm chooses the highest quality. The second firm is then forced to choose higher quality in order to narrow the gap between the first and the second firm. In Phase 5, when the fixed cost is very small, there are multiple equilibria and the first entrant chooses middle quality in some of them (Hence its quality may fall as the fixed cost decreases).

The above result indicates that everyone becomes worse off as the fixed cost decreases when the first entrant deters entry of the second firm (i.e. in Phase 2). As the fixed cost decreases, the first entrant cuts down the quality of its product and each customer obtains less utility from consuming it. Therefore both of the first entrant’s profit and consumers’ utilities decrease. This argument demonstrates that a policy intervention that enhances competition is harmful to the society when no entry occurs afterwards.

There are a considerable number of studies about product-line selection in a differentiated oligopoly market. We can classify them into three types. The studies of the first type assume that two firms choose their product varieties simultaneously. Gal-Or (1983) and Wernerfelt (1986) investigate the optimal product line when firms compete in quantity. Brander and Eaton (1984) analyze the optimal product choice when each firm chooses two varieties. Martinez-Giralt and Neven (1988) consider the horizontal differentiation model of d’Aspremont, Gabszewicz, and Thisse (1979) (Hotelling (1929)’s model with quadratic transportation costs: we call it ‘Horizontal model’ hereafter) and show that firms choose only one product each even if they can select any number. Martinez-Giralt (1989) obtains the same result in the vertically differentiated Hotelling model with quadratic transportation costs. Champsaur and Rochet (1989) and Cremer and Thisse (1991) prove that the vertical differentiation model of Mussa and
Rosen (1978) is mathematically equivalent to Horizontal model if marginal cost is a quadratic function of the quality. Therefore both firms choose one product each in the model of Mussa and Rosen (1978). However, “the assumption of a quadratic transportation cost might be crucial for these results, since this function is known to generate most intense price competition” (Martinez-Giralt and Neven (1988), pp. 436-437). Furthermore, the studies of this type leave it out of account that an incumbent firm can choose its product line in advance of an entrant.

The papers of the second type assume that firms move sequentially. Schmalensee (1978) argues that an incumbent firm can successfully prevent entry by producing enough varieties. Bonanno (1987) uses Horizontal model and shows that an incumbent firm can stop entry by changing location of its products. Constantatos and Perrakis (1997) obtain a similar result in the vertical differentiation model of Gabszewicz and Thisse (1979). Nevertheless, these papers have two shortcomings: they cannot explain why monopoly is rare in reality, and they do not check the credibility of the incumbent’s strategy.

The papers of the third type explicitly deal with the commitment problem of an incumbent after entry occurs. Judd (1985) shows with a two-goods model that, if an exit cost is small, the incumbent cannot stop entry by choosing both goods since it has an ex post incentive to withdraw the product entry occurs. It is not clear, however, whether his argument can be extended to a multi-product market, since the incumbent might cluster its products and keep all of them except the one that directly competes with the entrant. Ashiya (2000) proves that choosing two or more products does not help the incumbent deter entry in Horizontal model, but he also assumes a quadratic transportation cost.

This paper reveals three important points that have not appeared in the literature. First, it shows without help of the quadratic transportation costs (or the marginal cost that is a quadratic function of the quality) that brand proliferation is useless for an incumbent to prevent entry. Secondly, this paper is the first attempt in a vertically differentiated market to consider the sequential location problem of more than two firms, all of which can choose any variety of products. It finds that only the second entrant changes its quality (to narrow the distance from the first entrant and deter entry of the third firm) when the fixed cost decreases in the middle range. Thirdly, this paper derives
the social welfare function explicitly, and shows the exact range of the fixed cost in which the social welfare is increasing in the fixed cost.\textsuperscript{5}

The paper is organized as follows. Section 2 describes the basic model. Section 3 shows that in equilibrium the first (second) entrant withdraws products near to the second (first) entrant if it has chosen more than one product. Section 4 investigates the optimal product of the first entrant given the fixed cost. Section 5 considers the extended model where three firms move sequentially, and Section 6 analyzes the social welfare. Section 7 concludes this paper. The Appendix contains the formal proofs.

2. The basic model
We use the vertical differentiation model of Gabszewicz and Thisse (1979): A denotes the first mover and B denotes the second mover.\textsuperscript{6} The timing of each firm’s action, which is common knowledge, is as follows. At date 1, A chooses a set of products $Q_A = \{q_1, \ldots, q_n\}$ ($q_1 < \cdots < q_n$) from a technologically feasible range of qualities $[1, Q]$. At date 2, B observes $Q_A$ and chooses $Q_B = \{q_{B1}, q_{B2}, \ldots, q_{Bm}\}$ ($q_{B1} < \cdots < q_{Bm}$). Each firm sinks a fixed cost $F$ per product if it enters the market. If firm $i$ does not enter the market, $Q_i = \emptyset$.

At date 3, each firm observes $(Q_A, Q_B)$ and simultaneously selects a set of products to withdraw.\textsuperscript{7} Since a firm can get out of the market at will in reality, each firm can withdraw its product(s) with no additional cost (It cannot recover the fixed cost).\textsuperscript{8} Let $\hat{Q}_i$ be the set of products firm $i$ does not withdraw ($\hat{Q}_i \subseteq Q_i$). If $\hat{Q}_i = \{q_i\}$, we write $\hat{Q}_i = q_i$.

At date 4, each firm observes $(\hat{Q}_A, \hat{Q}_B)$, and simultaneously selects prices of its products. Each pays variable costs and earns sales revenue. Each firm makes goods at constant marginal cost, which is assumed to be zero regardless of the quality.\textsuperscript{9}

Consumers are identical in tastes but differing in income. Their incomes are uniformly distributed on the segment $[1, h]$. Shaked and Sutton (1982) show that only two firms of the highest and the second highest quality can earn positive gross profit if $2 < h < 4$. Only one firm of the highest quality can earn positive profit if $h < 2$. Thus
we assume \( h = 3 \) (Consequently, consumers are distributed with density 0.5). Each consumer purchases one unit of the good for which her indirect utility is maximized, or buys nothing if it is better. The utility of a consumer of income \( y \in [1, 3] \) who bought \( q_i \) at price \( P_i \) is

\[
U(q_i; y) = q_i (y - P_i).
\]

If she bought nothing, her utility is

\[
U(0; y) = y. \tag{10}
\]

The equilibrium concept we adopt is a weak refinement of subgame-perfect Nash equilibrium that assumes no weakly dominated strategy is played in equilibrium. \(^{11}\) Let \( \Pi_i(\hat{Q}_A, \hat{Q}_B) \) be the equilibrium profit of firm \( i \) gross of the fixed cost. Entry occurs when \( \Pi_i \) is larger than the fixed cost.

3. The equilibrium profits

This section calculates the equilibrium profits after the subgame of date 3. First we consider the case that \( A \) and \( B \) choose one product each. Using the result of Gabszewicz and Thisse (1979), Lemma 1 shows that it is better for \( B \) to locate its product far apart from \( A \) to mitigate competition. (The proofs of all lemmata and propositions can be found in the Appendix).

Lemma 1.

(a) Suppose \( q_i < Q \). Then \( \Pi_B(q_i, Q) > \Pi_B(q_i, q_B) \) for any \( q_B \in [q_1, Q] \).

(b) Define

\[
q_B^*(q_i) = \begin{cases} 
0.25q_i + 0.75 & \text{for } q_i \leq 3 \\
-1 - q_i + (q_i - 1)/[1 + q_i] & \text{for } q_i > 3.
\end{cases}
\]

Then \( \Pi_B(q_i, q_B^*(q_i)) \geq \Pi_B(q_i, q_B) \) for any \( q_B \leq q_i \).

If \( q_i \leq q_B < Q \), increasing \( q_B \) raises \( \Pi_B \) because competition is mitigated and the quality is improved. Therefore the best choice is \( q_B = Q \). If \( q_B < q_1 \), on the other
hand, increasing $q_B$ intensifies competition, while decreasing $q_B$ deteriorates the quality. Therefore it is better to choose a middle quality.

Secondly, we consider the case that $A$ chooses $n$ ($\geq 2$) goods, $B$ chooses one good, and $B$’s product is the highest quality among them. Since we assume consumers’ incomes (i.e. willingness to pay for quality) are similar, everyone prefers ‘an expensive but high-quality good’ to ‘a cheap but low-quality good’ in this model. Accordingly, only two products, $B$’s product and $A$’s top-quality product, have positive sales in equilibrium on condition that the quality gap between them is not so large. 12 Furthermore, if $A$ keeps products near to $B$, it faces intense competition. Thus $A$ withdraws all products near to $B$ in equilibrium (Note that “keeping all products and charging very high prices near $B$” is not equal to “withdrawing products nearer to $B$”; $A$ cannot commit itself to the former strategy but can commit to the latter).

Lemma 2.
Suppose $A$ chooses $n$ ($\geq 2$) goods such that $q_1 < \cdots < q_{k-1} < q^*_B(Q) \leq q_k < \cdots < q_n$, $Q < 11 + 4\sqrt{6}(\approx 20.798)$, and $B$ chooses $q_B = Q$. Then in equilibrium,
(a) $A$ withdraws $\{q_{k+1}, \ldots, q_n\}$, keeps $q_k$, and earns $\Pi_A(q_k, Q)$, and $B$ earns $\Pi_B(q_k, Q)$ ($A$ may or may not withdraw $\{q_1, \cdots, q_{k-1}\}$) if $\Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q)$;
(b) $A$ withdraws $\{q_1, \cdots, q_n\}$, keeps $q_{k-1}$, and earns $\Pi_A(q_{k-1}, Q)$, and $B$ earns $\Pi_B(q_{k-1}, Q)$ ($A$ may or may not withdraw $\{q_1, \cdots, q_{k-2}\}$) if $\Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q)$.

Thirdly, Lemma 3 considers the case that $B$ chooses a low-quality good. If $A$ keeps its all products, it can separate the market and operate discrimination. The gain from it is small, however, because consumers’ tastes are similar in our model. Thus in equilibrium $A$ withdraws the products near to $B$ in order to avoid competition.

Lemma 3.
(a) Suppose $q_{n-1} \leq q_B < q_n$. Then $A$ earns $\Pi_A(q_n, q_B)$ and $B$ earns $\Pi_B(q_n, q_B)$ in equilibrium.
(b) Suppose \( q_{k-1} \leq q_B < q_k < \cdots < q_n \) and \( q_B \geq 0.25q_n + 0.75 \). Then A withdraws \( \{q_k, \ldots, q_{n-1}\} \), keeps \( q_n \), and earns \( \Pi_A(q_n, q_B) \), and B earns \( \Pi_B(q_n, q_B) \) in equilibrium.

By bringing Lemma 2 and Lemma 3 together, Proposition 1 shows the sufficient condition for B to enter the market when A chooses two or more products. It shows that choosing two or more kinds of products facilitates entry in the top quality.

Proposition 1.

Suppose A chooses \( n \ (\geq 2) \) goods such that \( q_{k-1} < q_B^*(Q) \leq q_k < \cdots < q_n \) and \( Q < 11 + 4\sqrt{6} \). Then B can enter the market if (a), (b), (c) or (d) is satisfied.

(a) \( \Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q) \) and \( \Pi_B(q_k, Q) > F \).

(b) \( \Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q) \) and \( \Pi_B(q_{k-1}, Q) > F \).

(c) \( \Pi_B(q_n, q_B) > F \) for some \( q_B \in [q_{n-1}, q_n] \).

(d) \( \Pi_B(q_n, 0.25q_n + 0.75) > F \).

Next we examine the maximum profit A can earn when it chooses two or more products and B enters the market. Lemma 4 shows that A’s profit weakly decreases as B chooses more products. Therefore we have only to consider the case that B chooses one product. Proposition 2 summarizes the results.

Lemma 4.

Suppose \( \hat{Q}_B \neq \phi \). Let us exclude \( q_{Bi} \) from \( \hat{Q}_B \) and define the remainder as \( \hat{Q}_B' \). Then \( \Pi_A(\hat{Q}_A, \hat{Q}_B) \leq \Pi_A(\hat{Q}_A, \hat{Q}_B') \) for any \( \hat{Q}_A \), \( \hat{Q}_B \), and \( q_{Bi} \).

Proposition 2.

Suppose A chooses \( n \ (\geq 2) \) goods such that \( q_{k-1} < q_B^*(Q) \leq q_k < \cdots < q_n \) and \( Q < 11 + 4\sqrt{6} \). Then in equilibrium A can earn at most
(a) $\Pi_A(q_k, Q)$ if $\Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q)$ and $\Pi_B(q_k, Q) \geq \max\{F, \Pi_B(q_n^*, q_n^*(q_n))\}$;

(b) $\Pi_A(q_{k-1}, Q)$ if $\Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q)$ and $\Pi_B(q_{k-1}, Q) \geq \max\{F, \Pi_B(q_n, q_n^*(q_n))\}$;

(c) $\Pi_A(q_n^*, q_n^*(q_n))$ if $\Pi_B(q_n, 0.25q_n + 0.75) > F$.

Finally, we consider $B$’s optimal product. Proposition 3 shows that $B$ has no incentive to choose two or more products when $A$ chooses one product. We can prove it using the corollaries of Lemma 2 and Lemma 3.

Corollary of Lemma 2.

Suppose $A$ chooses $q_1 < 11 + 4\sqrt{6}$ and $B$ chooses $n$ (≥ 2) goods such that $q_{B1} < \cdots < q_{Bk-1} < q_n^*(q_1) \leq q_{Bk} < \cdots < q_{Bm} \leq q_1$. Then in equilibrium,

(a) $B$ withdraws $\{q_{Bk+1}, \cdots, q_{Bm}\}$, keeps $q_{Bk}$, and earns $\Pi_B(q_1, q_{Bk})$ (It may or may not withdraw $\{q_{B1}, \cdots, q_{Bk-1}\}$ if $\Pi_B(q_{B1}, q_{Bk-1}) \leq \Pi_B(q_1, q_{Bk})$;

(b) $B$ withdraws $\{q_{Bk}, \cdots, q_{Bm}\}$, keeps $q_{Bk}$, and earns $\Pi_B(q_1, q_{Bk})$ (It may or may not withdraw $\{q_{B1}, \cdots, q_{Bk-2}\}$ if $\Pi_B(q_1, q_{Bk-1}) > \Pi_B(q_1, q_{Bk})$).

Corollary of Lemma 3.

Suppose $A$ chooses $q_1 \geq 0.25Q + 0.75$ and $B$ chooses $n$ (≥ 2) goods such that $q_{B1} < \cdots < q_{Bk-1} < q_1 \leq \cdots < q_{Bm}$. Then $B$ withdraws $\{q_{Bk}, \cdots, q_{Bm}\}$, keeps $q_{Bm}$, and earns $\Pi_B(q_1, q_{Bm})$ in equilibrium.

Proposition 3.

Suppose $Q < 11 + 4\sqrt{6}$ and $A$ chooses $q_1 \geq 0.25Q + 0.75$. Then $B$ chooses one product at most in equilibrium.

4. The optimal product of the first entrant

The last section proved that the first entrant faces the commitment problem if it has two
or more products. It cannot commit to keep them after subsequent entry occurs, since withdrawal of competing products relaxes competition and raises profits from the remaining products. This section demonstrates that the first entrant always chooses one product in equilibrium. Figure 1 indicates the optimal product of the first entrant as a function of the fixed cost.

Proposition 4 shows that $A$ chooses the product of the highest quality when the fixed cost is large enough to blockade entry. If there were no other firm, $A$ could choose both a high-quality product and a low-quality product and screen its customers. Faced with a potential entrant, however, $A$ must cluster its products in order to deter entry in the top quality. Thus the additional profit $A$ can earn from the second product is so small that $A$ cannot recover its fixed cost (The proofs can be found in the Appendix).

**Proposition 4 (Phase 1: Blockaded entry).**

Suppose $Q < 11 + 4\sqrt{6}$ and $F \geq \Pi_B(Q, q^*_B(Q))$. Then $A$ chooses $Q_A = Q$ and $B$ does not enter the market in the unique equilibrium.

Proposition 5 investigates the case that $A$ cannot deter entry by choosing the highest quality. It shows that, in order to prevent entry of a low-quality firm, $A$ degrades the quality of its product as the fixed cost decreases. Since we cannot obtain the closed-form solution of $q^*$ (defined in Lemma 5 below), we prove Proposition 5 and 6 by numerical calculation. The outline of the proof is as follows. First we calculate $A$’s equilibrium profit. Next we show $A$’s profit decreases when it deviates from the equilibrium strategy. We have only to consider the following three cases; (a) $A$ chooses one product, (b) $A$ chooses two or more products and $B$ enters the market, and (c) $A$ chooses two or more products and $B$ does not enter the market.

**Lemma 5.**

Define $q^*$ such that

$$\Pi_B(q^*, Q) = \Pi_B(q^*, q^*_B(q^*))$$

Then $A$ can deter entry by choosing one product if and only if $F \geq \Pi_B(q^*, Q)$.
Proposition 5 (Phase 2: Deterred entry).

Suppose \( Q < 11 + 4\sqrt{6} \) and \( \Pi_B(q^*, Q) \leq F < \Pi_B(Q, q_B^*(Q)) \). Define \( \bar{q}_i \) such that

\[
\Pi_B(\bar{q}_i, q_B^*(\bar{q}_i)) = F.
\]

Then \( A \) chooses \( Q_A = \bar{q}_i \) and \( B \) does not enter the market in the unique equilibrium.

When the fixed cost is too small to deter entry, Proposition 6 shows that \( A \) chooses the highest quality to secure its profit. The proof goes through the same steps as that of Proposition 5.

Proposition 6 (Phase 3: Allowed entry).

Suppose \( Q < 11 + 4\sqrt{6} \) and \( F < \Pi_B(q^*, Q) \). Then \( A \) chooses \( Q_A = Q \) and \( B \) chooses \( Q_B = q_B^*(Q) \) in the unique equilibrium.

The combination of Proposition 4, 5, and 6 yields Theorem 1: the first entrant always chooses one product. Calculation shows that \( A \) would choose two products if there were no other firm and \( F < \Pi_A(\sqrt{Q}, Q, \phi) \). Therefore the incumbent firm stops proliferating its brand when there is a potential entrant.

Theorem 1.

\( A \) chooses one product in equilibrium regardless of \( F \).

Corollary of Theorem 1.

The number of products \( A \) chooses when faced with a potential entrant is equal to or smaller than that in the absence of other firms.

This result presents a striking contrast to the argument of Schmalensee (1978), which suggests that an incumbent firm can prevent entry by filling up the product spectrum. The difference comes from the following point: Schmalensee does not pay
proper attention to the commitment problem the incumbent faces, while this paper addresses this problem and shows that the incumbent’s threat of tough competition is incredible. When the incumbent fills up the market and entry occurs afterwards, the incumbent withdraws products near to the entrant to relax competition and secure its profits from other products. Since the entrant foresees it and enters any place it wants to be, brand proliferation is the *worst* strategy for the incumbent firm.

5. Extension

This section extends the model and assumes that the third firm, $C$, moves after firm $B$. In this extended model, firm $B$ maximizes its profit subject to the constraint that it must deter entry of firm $C$ given firm $A$’s product. Firm $A$ then chooses its product taking account of $B$’s response. See Figure 2.

When the fixed cost is large, entry of $C$ is blockaded and $(q_A, q_B) = \left( Q, q_B^*(Q) \right)$ (Phase 3). As the fixed cost decreases, $A$ and $B$ must choose closer qualities in order to deter entry of $C$ between them. Given $q_A$, $B$ chooses the quality just enough to deter entry (to mitigate competition with $A$). When the fixed cost is relatively large, $A$ continues choosing the top quality and $B$ is forced to choose higher quality (Phase 4). In Phase 5, when the fixed cost becomes very small, there are multiple equilibria. Since we assume income dispersion is small, firm $C$ cannot enter the market for any positive fixed cost.\(^\text{13}\)

Proposition 7.

Suppose $Q < 11 + 4\sqrt{6}$ and the third firm, $C$, moves after firm $B$. Let $\Pi_i(q_A, q_B, q_C)$ be the profit function of firm $i$. Then

(a) $C$ never enters the market in equilibrium.

(b) $(q_A, q_B) = \left( Q, q_B^*(Q) \right)$ in the unique equilibrium if

$$\Pi_c \left( Q, q_B^*(Q), 0.4Q + 0.6q_B^*(Q) \right) \leq F < \Pi_b \left( q^*, Q, \phi \right).$$

(c) $(q_A, q_B) = \left( Q, \bar{q}_B \right)$ such that $\Pi_c \left( Q, \bar{q}_B, 0.4Q + 0.6\bar{q}_B \right) = F$ in the unique equilibrium if $\Pi_c \left( Q, 0.25Q + 0.75, 0.55Q + 0.45 \right) \leq F < \Pi_c \left( Q, q_B^*(Q), 0.4Q + 0.6q_B^*(Q) \right)$.\(^{13}\)
(d) Define 
\[ q_{bet} = \left\{ q_A \mid \Pi_C(q_{bet}, 0.25Q + 0.75, 0.4q_{bet} + 0.15Q + 0.45) = F \right\}, \]
\[ q_{up} = \left\{ q_A \mid \Pi_B(q_{up}, Q, \phi) = F \right\}, \]
\[ F^* \equiv \left\{ F \mid q_{bet} = q_{up} \right\}. \]

Then in equilibrium 
(d1) A chooses \( q_A \in [q_{bet}, Q] \) if \( F^* \leq F < \Pi_C(Q, 0.25Q + 0.75, 0.55Q + 0.45) \);
(d2) A chooses \( q_A \in [q_{up}, Q] \) if \( F < F^* \); and
(d3) B chooses \( q_B \) such that \( q_B < q_A \) and \( \Pi_C(q_A, q_B, 0.4q_A + 0.6q_B) = F \) if
\[ F < \Pi_C(Q, 0.25Q + 0.75, 0.55Q + 0.45). \]

Proposition 7 (b), (c), and (d) correspond to Phase 3, 4, and 5 in Figure 2 respectively. Entry of C is blocked in Phase 3. A and B must choose closer qualities to deter entry of C in Phase 4 and 5. Since B wants to mitigate competition with A, it chooses the quality just enough to deter entry of C given A’s product. In Phase 4, A always chooses the highest quality, and forces B to bear the burden of entry deterrence. In Phase 5, A chooses the product from the shaded region of Figure 2. The lowest quality it chooses in equilibrium is determined by two conditions. The first condition is technical one. Calculation shows that it is indifferent for A to choose any quality as far as B chooses \( q_B \geq 0.25q_A + 0.75 \) and entry of C is deterred. The lowest quality that satisfies these conditions is \( q_{bet} \). \( q_{bet} \) is increasing in \( F \) because (in order to deter entry of C) A must choose closer product to B as the fixed cost decreases. The second condition is that A must not allow entry of B in the highest quality. Therefore \( q_A \geq q_{up} \) must hold. Since \( q_{up} \) is decreasing in \( F \), A must choose its quality from narrower range when the fixed cost becomes sufficiently small.

Let us discuss Phase 4 in detail. A and B must choose similar products and commit themselves to tough competition when entry threat is severe. A can deter entry by either downgrading its product or keeping its quality and forcing B to upgrade. The former strategy secures its customers, but the customers will pay less for its now deteriorated
product. The latter strategy has the opposite merit and demerit. Which strategy is better for firm A, then? Proposition 7 finds that A chooses the latter strategy in Phase 4. Accordingly, only the second entrant (firm B) changes its quality when the fixed cost decreases in Phase 4.

This result is in sharp contrast to Ashiya (2000), which finds that only the first entrant changes its location in Horizontal model. The intuition is as follows. Keeping the same location has no merit in a horizontally differentiated market. On the contrary, this strategy reduces the market of the first entrant because the second firm narrows the distance between them (in order to deter entry of the third firm). Therefore it is better for the first entrant to move inward and secure customers in Horizontal model. These findings demonstrate the essential difference between the location problem in the vertically differentiated market and that in the horizontally differentiated market.

6. Welfare analysis

This section investigates how the social welfare changes as the fixed cost decreases. The change can be divided into five phases, and Theorem 2 below shows that the decrease of the fixed cost makes everyone (weakly) worse off in the second phase, i.e., when the first firm deters entry of the second firm.

We face the following problems when we quantify the welfare change. Surplus maximization is not equal to Pareto efficiency since preferences are not quasi-linear. A utilitarian welfare function is not a good measure either, because we cannot determine the appropriate weight to the profits in our partial analysis. Accordingly we use the compensating variations as the welfare measure, although it implies a social welfare function whose weight to each consumer is inversely proportional to his marginal utility of income.

More specifically, we regard the market with no product as being the status quo and calculate the compensating variations. The social welfare, $W$, is defined as the sum of the compensating variations and each firm’s net profit. Suppose $A$, $B$, and $C$ initially choose $s$ products in all (Some of them may be withdrawn afterwards). Let us rename them $q_1 < \cdots < q_s$, and let $[l_i, h_i]$ be the income range of the consumers who buy product $q_i$. 


\( l_i = h_i \) if no one buys \( q_i \) or \( q_i \) is withdrawn. Then
\[
W = 0.5 \sum_{i=1}^s \int_{l_i}^{h_i} y(1 - q_i^{-1})dy - sF
\]
(Remember that consumers are distributed with density 0.5) and we can draw Figure 3.

When the fixed cost is so large that entry is blockaded, the first entrant always chooses the same quality and price (Proposition 4). Thus each consumer obtains the same utility, and the net profit of the first firm increases by the same amount as the decrease of the fixed cost (Phase 1).

When the fixed cost becomes small and entry of the second firm is not blockaded, Proposition 5 has shown that the first firm degrades the quality as the fixed cost decreases (Phase 2). Then calculation shows the following: (1) the net profit of the first firm decreases, (2) the market served by the first firm does not change, and (3) each buyer obtains lower utility. Therefore everyone becomes worse off in this phase as the fixed cost decreases (i.e. as the threat of the entrant is strengthened).

Theorem 2.

The equilibrium outcome under \( F = F' \) is Pareto superior to the equilibrium outcome under \( F = F'' \) if \( \Pi_b(q^*_b, Q, \phi) \leq F'' < F' \leq \Pi_b(Q, q^*_b(Q, \phi)) \) and \( Q < 11 + 4\sqrt{6} \).

Proof of Theorem 2.

Suppose \( \Pi_b(q^*, Q, \phi) \leq F \leq \Pi_b(Q, q^*_b(Q, \phi)) \) and \( Q < 11 + 4\sqrt{6} \). Then Proposition 5 shows that in equilibrium \( A \) offers \( \bar{q}_i \) that satisfies
\[
F = \Pi_b(\bar{q}_i, q^*_b(\bar{q}_i), \phi) = \frac{(\bar{q}_i - 1)(5\bar{q}_i + \bar{q}_i r - 4\bar{q}_i - 4)}{4(\bar{q}_i - r - 1)(\bar{q}_i r - \bar{q}_i - r - 1)} \quad \text{where} \quad r = \sqrt{\bar{q}_i} + 1.
\]

Note that \( \frac{d\bar{q}_i}{dF} > 0 \). The equilibrium price is \( P_i = \frac{3(q_i - 1)}{2q_i} \), and \( A \)'s net profit is
\[
\Pi_A - F = \frac{9(q_i - 1)}{8\bar{q}_i} - \frac{(q_i - 1)(5\bar{q}_i + \bar{q}_i r - 4\bar{q}_i - 4)}{4(\bar{q}_i - r - 1)(\bar{q}_i r - \bar{q}_i - r - 1)}.
\]
Since calculation shows that $\frac{d}{dq_i} [\Pi_A - F] > 0$, firm A becomes worse off as $F$ decreases.

The utility a consumer of income $y$ obtains is

$$U(q_i; y) = \begin{cases} 
q_i(y - 1.5) + 1.5 & \text{if } 1.5 \leq y \leq 3 \\
y & \text{otherwise}
\end{cases}$$

(A consumer whose income is less than 1.5 buys nothing). Thus consumers whose incomes are larger than 1.5 become worse off as $F$ decreases. The change of $F$ has no effect on the poor consumers and other firms. Q.E.D.

In Phase 3, when the fixed cost is so small that the first firm cannot deter entry of the second firm, Proposition 6 has shown that each firm chooses the fixed product ($Q_A = Q$ and $Q_B = q_B^*(Q)$). Consequently the social welfare increases by the same amount as the decrease of the fixed costs (of two firms). The social welfare under duopoly is larger than that under monopoly because competition drives the prices down and more people buy the top quality good.

When the fixed cost decreases still further, the first and the second firm must choose similar qualities to deter entry of the third firm between them. In Phase 4, when the fixed cost is relatively large, Proposition 7 (c) has shown that the first firm keeps the highest quality and forces the second firm to choose higher quality. Therefore the social welfare increases more than the amount the fixed costs decrease. In Phase 5, when the fixed cost is very small, the first entrant chooses its quality from the range specified in Proposition 7 (d). The social welfare may increase or decrease as the fixed cost decreases, since the first firm may choose higher or lower quality as the fixed cost decreases.

Proposition 8.

Define $W$ as the sum of the compensating variations and each firm’s net profit. Then

(a) $\frac{dW}{dF} = -1$ if $F \geq \Pi_B(Q, q_B^*(Q), \phi)$;

(b) $\frac{dW}{dF} > 0$ if $\Pi_B(q^*, Q, \phi) \leq F < \Pi_B(Q, q_B^*(Q), \phi)$:
(c) \( W \) evaluated at \( F = \Pi_B(q^*,Q,\phi) - \varepsilon \) is larger than \( W \) at \( F = \Pi_B(q^*,Q,\phi) \);

(d) \( \frac{dW}{dF} = -2 \) if \( \Pi_c(Q,q^*_b(Q),0.4Q + 0.6q^*_b(Q)) \leq F < \Pi_B(q^*,Q,\phi) \);

(e) \( \frac{dW}{dF} < -2 \) if \( \Pi_c(Q,0.25Q + 0.75,0.55Q + 0.45) \leq F < \Pi_c(Q,q^*_b(Q),0.4Q + 0.6q^*_b(Q)) \);

(f) \( W \in [W_L,W_H] \) if \( F < \Pi_c(Q,0.25Q + 0.75,0.55Q + 0.45) \), and

\( (f1) \frac{dW_L}{dF} < -2 \) for \( F < \Pi_c(Q,0.25Q + 0.75,0.55Q + 0.45) \),

\( (f2) \frac{dW_L}{dF} > 0 \) for \( F^* \leq F < \Pi_c(Q,0.25Q + 0.75,0.55Q + 0.45) \), and

\( (f3) \frac{dW_L}{dF} < -2 \) for \( F < F^* \).

Proposition 8 warns that a policy that lowers entry barrier reduces the social welfare if it fails to establish the second firm in this market. Suppose the government deregulates the monopolized market and tries to enhance competition. If the new entrant is not strong enough, the incumbent firm deters it by cutting down the quality of its product and reducing the market niche. This brings about a negative effect on the social welfare. Although the government can help the entrant, a small amount of subsidy makes matters worse. The subsidy (or deregulation) must be large enough so that the newcomer can enter the market successfully.

7. Conclusions

After entry occurs, it may be profitable for an incumbent firm to withdraw products near to the entrant and relax competition. We have explicitly dealt with this commitment problem, and have proved that choosing two or more kinds of products cannot deter entry in a vertically differentiated market. The entry threat causes the incumbent firm to avoid brand proliferation: the incumbent always chooses one good in equilibrium.

When the incumbent deters entry, it degrades quality and consequently everyone becomes worse off as the entrant becomes dangerous. Hence a competition-enhancing policy such as deregulation or subsidy to the entrant is harmful if entry is unsuccessful.
The policy must be comprehensive enough for the newcomer to enter the market successfully in order to improve the social welfare.
Notes

1. This paper assumes income dispersion of consumers is so small that the third entrant cannot enter the market in equilibrium for any positive fixed cost.

2. The social welfare decreases as the fixed cost decreases in some equilibria of Phase 5 too.

3. It assumes that consumers are located on $[0,1]$ and products are located on $[1,\infty)$.

4. Neven (1987) simulates a sequential entry of identical firms in the Horizontal model, but it assumes that each firm can choose one product only and that the exit cost is infinite. Ashiya (2000) considers the optimal product variety of three firms in Horizontal model.

5. Constantatos and Perrakis (1997) argue that the social welfare decreases as the fixed cost decreases if the incumbent deters entry by product relocation. However, they ignore the commitment problem of the incumbent firm, and they do not show the explicit conditions for the social welfare to increase in the fixed cost. Constantatos and Perrakis (1999) consider a sequential entry of three firms in a vertically differentiated market. They assume that a marketing cost is required (after a firm chooses its product and sinks a fixed cost) to put a product in the market. They show that a protected monopoly may be welfare superior to a duopoly when the fixed cost function (of quality) has some special properties. Their assumption about the marketing cost implies, however, that the second entrant in their model produces nothing if the third entrant, after the second firm sinks the fixed cost, chooses a higher quality than the second firm does. Their argument relies on this unrealistic assumption.

6. Section 5 considers the extended model with three firms.

7. Judd (1985 p.156 fn. 3) argues that “this is the correct static approximation of a truly dynamic analysis of entry into a growing market. Intuitively, in a continuous-time analysis, no one firm can commit itself to staying since tomorrow will give another chance to exit.”

8. An exit cost is the cost arising only because of the act of exit. One example is a printing cost of a new catalogue (from which withdrawn products are deleted). Note
that irreversible investment in product-specific capital is a sunk cost and is not an exit cost.

9. We can relax this assumption to the degree that the “finiteness property”, which Shaked and Sutton (1983) define, is satisfied. Namely, the argument of this paper can be applied to the case that the marginal cost rises with quality, as far as the speed is so slow that everyone prefers the same product when each product is sold at its marginal cost.

10. This model differs from Horizontal model in that all consumers choose the product of the highest quality when each product is sold at its marginal cost.

11. Consider the subgame where the first entrant chooses two products and the second entrant chooses one product that is the same quality as one of the first entrant’s product. If an exit cost is zero, it is also a subgame-perfect Nash equilibrium after this history that the second entrant exits and the first entrant keeps all products at date 3. We need further refinement to exclude this unrealistic equilibrium.

12. Lemma 2 below covers the case of \( Q < 11 + 4\sqrt{6} (\approx 20.798) \). If \( Q \geq 11 + 4\sqrt{6} \), the product of the highest quality is more than twenty times as valuable as that of the lowest quality. Then the market is so heterogeneous that the low-quality market is virtually separated from the high-quality one (where \( B \) enters). Therefore, when \( Q \geq 11 + 4\sqrt{6} \) and all of \( A \)’s products are chosen from the low-quality market, competition between \( A \) and \( B \) is mild enough that \( A \) can exercise some monopoly power (and may be able to screen the customers by its products). As a result, not only \( B \)’s product and \( A \)’s top-quality product but also \( A \)’s product of the second-highest quality may have positive sales in equilibrium. For example, both of \( A \)’s products have positive sales in equilibrium when \( (\hat{Q}_A, \hat{Q}_B) = (3, 4, 40) \). The profit functions in this case are too complicated to derive clear conclusions.

13. If consumers’ incomes were uniformly distributed on \([1, h]\) and \( h > 4 \), firm \( C \) would be viable and \( q_c < q_B < q_A = Q \) in equilibrium for a sufficiently small fixed cost.

14. \( A \) is indifferent between these two strategies in Phase 5, because it can earn the same profit by either strategy.
An anonymous referee points them out and facilitates the arguments below.
Appendix: Formal proofs of lemmata.

Proof of Lemma 1.

Define \( y_B \) such that \( y_B = q_B(y_B - P_B) \). Then a consumer of income \( y_B \) is indifferent between buying nothing and buying \( q_B \) at \( P_B \), and a consumer whose income is larger than \( y_B \) prefers buying \( q_B \) at \( P_B \) to buying nothing. Similarly, define \( y_1 \) such that \( y_1 = q_1(y_1 - P_1) \), and define \( y_{1B} \) such that \( q_1(y_{1B} - P_1) = q_B(y_{1B} - P_B) \). Then a consumer of income \( y_1 \) (\( y_{1B} \)) is indifferent between buying \( q_1 \) at \( P_1 \) and buying nothing (buying \( q_B \) at \( P_B \)). Therefore, for \( q_B \leq q_1 \),

\[
\Pi_A(q_1, q_B, P_1, P_B) = \begin{cases} 
0.5P_1(3 - y_{1B}) & \text{if } y_1 \leq y_{1B} \leq 3 \text{ and } 1 \leq y_{1B} \\
0.5P_1(3 - y_1) & \text{if } y_{1B} < y_1 \leq 3 \text{ and } 1 \leq y_1 , \text{ and } \\
P_1 & \text{if } y_1 < 1 \text{ and } y_{1B} < 1 
\end{cases}
\]

\[
\Pi_B(q_1, q_B, P_1, P_B) = \begin{cases} 
0.5P_B(y_{1B} - y_B) & \text{if } 1 \leq y_B \leq y_{1B} \\
0.5P_B(y_{1B} - 1) & \text{if } y_B < 1 \leq y_{1B}
\end{cases}
\]

(Note that the density of consumers is 0.5 in our model).

From FOCs, the equilibrium in the pricing game at date 4 is

\[
(p_1^*, p_B^*) = \begin{cases} 
\left(\frac{3q_1 - 2q_B - 1}{2q_1}, \frac{q_B - 1}{q_B}\right) & \text{if } q_B \leq 0.25(q_1 + 3) \text{ and } \\
\left(\frac{5(q_1 - q_B)}{3q_1}, \frac{q_1 - q_B}{3q_B}\right) & \text{if } 0.25(q_1 + 3) \leq q_B \leq q_1.
\end{cases}
\]

After the same calculation for \( q_B > q_1 \), we obtain

\[
\Pi_B(q_1, q_B) = \begin{cases} 
\left(\frac{3q_B - 2q_1 - 1}{8q_B(q_B - q_1)}\right)^2 & \text{if } q_B \geq 4q_1 - 3; \\
\frac{25q_B - q_1}{18q_B} & \text{if } q_1 \leq q_B \leq 4q_1 - 3; \\
\frac{q_1 - q_B}{18q_B} & \text{if } 0.25(q_1 + 3) \leq q_B \leq q_1; \text{ and } \\
\frac{(q_B - 1)(q_1 - 2q_B + 1)}{4q_B(q_1 - q_B)} & \equiv \pi_1 \text{ if } q_B \leq 0.25(q_1 + 3).
\end{cases}
\]
Then
\[ \frac{\partial \pi_1}{\partial q_B} = \frac{q_i^2 + q_1 - 2q_iq_B - q_iq_B^2 + 3q_B^2 - 2q_B}{4q_B(q_1 - q_B)^2} \]
and it is positive at \( q_B = 0.25(q_i + 3) \) if and only if \( q_i < 3 \). Thus it is straightforward to show (a) and (b). Q.E.D.

Proof of Lemma 2.

\( B \) always keeps \( q_B = Q \) and charges \( P_B > 0 \) because it is the weakly dominant strategy. Define \( q_Z = \max\{q_i|q_i| \in \hat{Q}_{A}\} \) and \( q_{Z-1} = \max\{q_i|q_i| \in \hat{Q}_{A}\ \backslash \ q_Z\} \). Calculation shows that the necessary condition for \( q_Z \) and \( q_{Z-1} \) to have positive sales is
\[ \frac{-2q_Z + q_{Z-1}(-3 + 4q_Z - Q) + 2Q}{q_{Z-1}(3q_{Z-1} + q_Z - 4Q)} > 0. \]
The numerator is minimum at \( q_Z = q_{Z-1} \), but \( 4q_{Z-1} - (5 + Q)q_{Z-1} + 2Q \) is positive because \( Q < 11 + 4\sqrt{6} \). Therefore \( q_B \) and \( q_Z \) only have positive sales and
\[ \Pi_A(q_A,q_B) = \Pi_A(q_Z,Q) = \left\{ \begin{array}{ll} (Q - q_Z) & \text{if } q_Z \geq 0.25(Q + 3) \\ \frac{18q_Z}{(q_Z - 1)(1 + Q - 2q_Z)} & \text{otherwise} \\ 4q_Z(Q - q_Z) & \end{array} \right. \]
in equilibrium. Since \( \Pi_A(q_Z,Q) \) is increasing in \( q_Z \) for \( q_Z < q_b^*(Q) \) and decreasing in \( q_Z \) for \( q_Z \geq q_b^*(Q) \), \( A \) should choose either \( q_Z = q_{k-1} \) or \( q_Z = q_k \). If \( \Pi_A(q_{k-1},Q) \leq \Pi_A(q_k,Q) \), \( A \) sets \( q_Z = q_k \) and earns \( \Pi_A(q_k,Q) \). If \( \Pi_A(q_{k-1},Q) > \Pi_A(q_k,Q) \), \( A \) sets \( q_Z = q_{k-1} \) and earns \( \Pi_A(q_{k-1},Q) \). Q.E.D.

Proof of Lemma 3 (a).

Suppose \( A \) keeps some products smaller than \( q_B \). Define
\[ q_X = \max\{q_i|q_i| \in \hat{Q}_{A}\ \text{a n d } q_i \leq q_B\}. \]
If \( q_B \leq 0.25(3q_X + q_n) \), then \( P_B = \frac{q_B - q_X}{q_B} \), \( P_n = \frac{3q_n - 2q_B - q_X}{2q_n} \), and \( P_i = 0 \) for
any $i \neq \{n, B\}$ in equilibrium. Hence $\Pi_A(Q_A, q_B) = \frac{(3q_n - 2q_B - q_X)^2}{8q_n(q_n - q_B)}$. Since Lemma 1 shows that

$$
\Pi_A(q_n, q_B) = \begin{cases}
\frac{25(q_n - q_B)}{18q_n} & \text{if } q_B \geq 0.25(q_n + 3), \\
\frac{(3q_n - 2q_B - 1)^2}{8q_n(q_n - q_B)} & \text{otherwise}
\end{cases}
$$

the best $A$ can do is to withdraw all products except $q_n$.

If $q_B > 0.25(3q_X + q_n)$ and $A$ keeps $q_n$, $A$’s products smaller than $q_B$ have no effect on $B$’s best response. $B$ chooses $P_B = \frac{q_n - q_B}{3q_B}$ and $A$ earns $\Pi_A(q_n, q_B)$ whether or not $A$ keeps some products smaller than $q_B$. Q.E.D.

Proof of Lemma 3 (b)

If $A$ keeps some $q_i \leq q_B$, $P_i = 0$ in equilibrium and $A$ earns no gross profit from them. Therefore $A$ withdraws $\{q_i, \ldots, q_{k-1}\}$ if they produce a negative effect on $P_B$. Otherwise $A$ may keep some of them.

Next we show $A$ has an incentive to withdraw $\{q_k, \ldots, q_n-1\}$. First suppose that $A$ keeps $q_k$ and $q_{k+1}$. Then in equilibrium

$$
P_B = 0.5q_B R^{-1}(2q_k^3 - q_kq_B(q_k + q_{k+1}) + q_B^2(q_{k+1} - q_k)),
$$

$$
P_k = R^{-1}(3q_k + 2q_{k+1})(q_k - q_B),
$$

$$
P_{k+1} = 0.5R^{-1}(2q_k(q_k + 4q_{k+1}) + q_B(q_{k+1} - 11q_k)),
$$

and $\Pi_A((q_k, q_{k+1}), q_B) = 0.125R^2V$,

where $R \equiv q_k^2 + 2q_kq_{k+1} - q_kq_B + q_{k+1}q_B$ and $V \equiv 12q_k^4 + q_k^2(40q_kq_{k+1} - 42q_B) + 2q_B^2(24q_k^2 - 43q_{k+1}q_B + 15q_B^3)$

$$
+ 7q_kq_{k+1}q_B(4q_k - 5q_B) + 5q_{k+1}q_B^2.
$$

Define $\Delta \Pi = \Pi_A((q_k, q_{k+1}), q_B) - \Pi_A(q_{k+1}, q_B)$. Then calculation shows that
\[
\frac{\partial^4}{\partial q_k^4} \Delta \Pi = 96(2q_{k+1} + 25q_B) > 0,
\]
\[
\frac{\partial^3}{\partial q_k^3} \Delta \Pi = -240q_{k+1}^2 + 1524q_{k+1}q_B + 1200q_B^2 > 0 \quad \text{at} \quad q_k = q_B,
\]
\[
\frac{\partial^2}{\partial q_k^2} \Delta \Pi = 64q_{k+1}^3 - 588q_{k+1}q_B + 1368q_{k+1}^2q_B + 200q_B^3 > 0 \quad \text{at} \quad q_k = q_B,
\]
\[
\Delta \Pi = -171q_{k+1}^2(q_{k+1} - q_B) < 0 \quad \text{at} \quad q_k = q_B, \quad \text{and}
\]
\[
\Delta \Pi = 0 \quad \text{at} \quad q_k = q_{k+1}.
\]
Consequently \( \Delta \Pi < 0 \) is always satisfied and A’s profit increases by withdrawal of \( q_k \).

Secondly, suppose \( A \) keeps \( n \) products. Then \( \Pi_A(\{q_k, \ldots, q_n\}, q_B) - \Pi_A(q_n, q_B) \) is decreasing in \( q_n \) for given \( q_B \). The reason is that a change in \( q_n \) has a direct effect on \( P_B \) in the latter case, while it has an indirect effect on \( P_B \) in the former case. Hence
\[
\Pi_A(\{q_k, \ldots, q_{n-1}, q_n\}, q_B) - \Pi_A(q_n, q_B)
\]
\[
< \Pi_A(\{q_k, \ldots, q_{n-1}, q_{n-1}\}, q_B) - \Pi_A(q_{n-1}, q_B)
\]
\[
= \Pi_A(\{q_k, \ldots, q_{n-1}\}, q_B) - \Pi_A(q_{n-1}, q_B)
\]
\[
< \Pi_A(\{q_k, \ldots, q_{n-2}\}, q_B) - \Pi_A(q_{n-2}, q_B)
\]
\[
< \cdots < \Pi_A(\{q_k, q_{k+1}\}, q_B) - \Pi_A(q_{k+1}, q_B) = \Delta \Pi < 0
\]
for given \( q_B \). Namely A’s profit increases by withdrawal of \( \{q_k, \ldots, q_{n-1}\} \). Q.E.D.

Proof of Lemma 4.

\( \Pi_A \) weakly increases if the competing product is withdrawn. Q.E.D.

Proof of Lemma 5.

Note that \( \Pi_B(q_1, Q) > \Pi_B(q_1, q^*_B(q_1)) \) at \( q_1 = 0.25Q + 0.75 \). Proposition 3 shows that \( B \) never chooses two or more products when \( A \) chooses \( q_1 \geq 0.25Q + 0.75 \). If \( B \) chooses one product, Lemma 1 shows that \( \Pi_B(q_1, q_B) \) is maximized at either \( q_B = Q \) or \( q_B = q^*_B(q_1) \). Hence choosing \( q^* \) deters entry if \( F \geq \Pi_B(q^*, Q) \). If \( F < \Pi_B(q^*, Q) \),
Proof of Proposition 1.

Suppose \( B \) enters \( q_B = Q \). Then Lemma 2 shows that it earns \( \Pi_B(q_B, Q) \) if \( \Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q) \) and it earns \( \Pi_B(q_{k-1}, Q) \) if \( \Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q) \).

Next suppose \( B \) enters \( q_B \geq q_{n-1} \). Then Lemma 3 (a) shows that it earns \( \Pi_B(q_n, q_B) \) in equilibrium.

Finally, suppose \( B \) enters \( q_B = 0.25q_n + 0.75 \). Then Lemma 3 (b) shows that it earns \( \Pi_B(q_n, 0.25q_n + 0.75) \) in equilibrium. Q.E.D.

Proof of Proposition 2.

Since Lemma 4 shows that \( A \)’s profit decreases as \( B \) chooses more products, we assume \( B \) chooses one product. If \( \Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q) \) and \( \Pi_B(q_k, Q) \geq \max \{F, \Pi_B(q_n, q_B^*(q_n))\} \), Lemma 2 (a) shows that \( B \) enters \( q_B = Q \) and \( A \) earns \( \Pi_A(q_k, Q) \). If \( \Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q) \) and \( \Pi_B(q_{k-1}, Q) \geq \max \{F, \Pi_B(q_n, q_B^*(q_n))\} \), Lemma 2(b) shows that \( B \) enters \( q_B = Q \) and \( A \) earns \( \Pi_A(q_{k-1}, Q) \). If \( \Pi_B(q_n, 0.25q_n + 0.75) > F \), Lemma 3 shows that \( B \) enters \( q_B \in [q_B^*(q_n), 0.25q_n + 0.75] \) such that \( A \) withdraws \( \{q_k, \ldots, q_{n-1}\} \) (or \( q_B = Q \) if it is better). Then the maximum profit \( A \) can earn is \( \Pi_A(q_n, q_B^*(q_n)) \). Q.E.D.

Proof of Proposition 3.

It is clear from Corollary of Lemma 2 and Corollary of Lemma 3 that \( B \)’s gross profit does not increase by choosing two or more product at date 2. Q.E.D.

Proof of Proposition 4.

First we calculate \( A \)’s equilibrium profit. If \( Q_A = Q \), \( B \) does not enter the market because Lemma 1 and Proposition 3 show that it can earn \( \Pi_B(Q, q_B^*(Q)) - F \) at most.
Hence $A$ earns $\Pi_A(Q, \phi) - F$ in equilibrium.

Next we show $A$ has no incentive to deviate from the equilibrium strategy. If $Q_A = q_i < Q$, $\Pi_A$ decreases because

$$\Pi_A(q_i, \phi) = 1.125(1 - q_i^{-1}).$$

If $A$ produces the second product and $\Pi_B(q_1, Q) > F$, Proposition 1 shows that $B$ enters $q_B = Q$ and Proposition 2 shows that $A$ earns $\Pi_A(q_1, Q) (< \Pi_A(Q, \phi))$.

If $A$ chooses $Q_A = \{q_1, q_2\}$ such that $\Pi_B(q_1, Q) \leq F$, calculation shows that

$$\Pi_A(q_1, q_2, \phi) = \frac{4.5q_1(q_2 - 1)}{q_2 + 3q_1q_2 - q_1 + q_1^2} < \Pi_A(Q, \phi) + \Pi_B(Q, q_B^*(Q)) \leq \Pi_A(Q, \phi) + F.$$  

Therefore $A$ never chooses two or more products. Q.E.D.

Proof of Proposition 5.

If $Q_A = \bar{q}_1$, $B$ does not enter the market and $A$ earns $\Pi_A(\bar{q}_1, \phi) - F$. We shall prove $\Pi_A$ decreases when $A$ deviates from this. First suppose that $A$ chooses one product other than $\bar{q}_1$. If $Q_A = q_1 < \bar{q}_1$, $\Pi_A$ decreases because $\Pi_A(q_1, \phi)$ is increasing in $q_1$. If $Q_A = q_1 > \bar{q}_1$, $B$ enters $q_B^*(q_1)$ because $\Pi_B(q_1, q_B^*(q_1)) > F$ and Proposition 3 shows that $B$ has no incentive to choose two or more products. Then $A$ earns $\Pi_A(q_1, q_B^*(q_1)) - F$ ($\leq \Pi_A(Q, q_B^*(Q)) - F$), and numerical calculation shows that $\Pi_A(Q, q_B^*(Q)) < \Pi_A(q_1, \phi) \leq \Pi_A(\bar{q}_1, \phi)$.

Next suppose that $A$ chooses $n \geq 2$ products and $q_k < q_B^*(Q) \leq q_k < \cdots < q_n$. Proposition 2 shows that $A$ can earn $\max \{\Pi_A(q_{k-1}, Q), \Pi_A(q_k, Q), \Pi_A(q_n, q_B^*(Q))\}$ at most when $B$ enters the market, and this profit is smaller than $\Pi_A(Q, q_B^*(Q))$. Therefore $A$ never chooses $n \geq 2$ products such that $B$ can enter the market afterwards.

Proposition 1 shows that, in order to deter entry of $B$, $q_k$ and $q_{k-1}$ must satisfy

$$\Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q).$$
and $\Pi_B(q_k, Q) \leq F \left( < \Pi_B(Q, q_B^*(Q)) \right)$.

Define $\bar{q}_k = \{q_k \mid \Pi_B(q_k, Q) = \Pi_B(Q, q_B^*(Q)) \}$. If $A$ chooses $q_k < \bar{q}_k$, $B$ can enter $q_B = Q$ because

$\Pi_B(q_k, Q) > \Pi_B(q, Q) > \Pi_B(\bar{q}_k, Q) = \Pi_B(Q, q_B^*(Q)) > F$. Hence $q_k \geq \bar{q}_k$ must hold. Next define $\bar{q}_{k-1} = \{q_{k-1} \mid \Pi_A(q_{k-1}, Q) = \Pi_A(\bar{q}_k, Q) \}$. Then calculation shows that

$\Pi_A(q, \phi) < \Pi_B(q^*, Q) (\leq F)$ for any $q \leq \bar{q}_{k-1}$.

Hence $A$ never chooses products smaller than $q_B^*(Q)$, and consequently $q_1 \geq \bar{q}_k$ must hold if $A$ chooses two or more products and deters entry of $B$. However,

$m_a \Pi_A(\{q_1, \ldots, q_{n-1}, q_1 \}, \phi) - m_a \Pi_A(\{q_1, \ldots, q_{n-1} \}, \phi) < m_a \Pi_A(\{q_1, \ldots, q_{n-2}, q_{n-1} \}, \phi) - m_a \Pi_A(\{q_1, \ldots, q_{n-2} \}, \phi),$

and numerical calculation shows that

$m_a \Pi_A(\{q_1, q_2, q_3 \}, \phi) = \Pi_A(\bar{q}_k, \sqrt{\bar{q}_k Q}, Q, \phi) < \Pi_A(\bar{q}_k, Q, \phi) + \Pi_B(q^*, Q) \leq \Pi_A(\bar{q}_k, Q, \phi) + F$

when $q_i \geq \bar{q}_k$. Namely, choosing three or more products decreases $A$’s net profit. Moreover, numerical calculation shows that

$\Pi_A(\bar{q}_k, Q, \phi) < \Pi_A(q^*, \phi) + \Pi_B(q^*, Q) \leq \Pi_A(q^*, \phi) + F \leq \Pi_A(\bar{q}_k, \phi) + F$.

Therefore choosing two products decreases $A$’s net profit. Q.E.D.

Proof of Proposition 6.

If $Q_A = Q$, Proposition 3 shows that $B$ chooses only one product, $q_B = q_B^*(Q)$, and $A$ earns $\Pi_A(Q, q_B^*(Q)) - F$. We shall prove that $\Pi_A$ decreases when $A$ deviates from this.

Suppose $A$ chooses $Q_A = q_1 < Q$. Then Lemma 1 and Proposition 3 show that $B$ enters either $q_b = Q$ or $q_b = q_b^*(q_1)$, and $A$ earns either $\Pi_A(q_1, Q) - F$ or $\Pi_A(q_1, q_b^*(q_1)) - F$. It is clear that they both are smaller than $\Pi_A(Q, q_B^*(Q)) - F$. 

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Next suppose that $A$ chooses $n \geq 2$ products and $q_{k-1} < q^*_b(Q) \leq q_k < \cdots < q_n$. First we consider the case that $F < \Pi_B(q^*, 0.25q^* + 0.75)$. If $q_k < q^*$, Proposition 1 shows that $B$ enters $q_b = Q$. Then Proposition 2 shows that $A$ can earn $\max \{ \Pi_A(q_{k-1}, Q), \Pi_A(q_k, Q) \} - nF$ at most. If $q_k \geq q^*$, $B$ enters the low quality market because Lemma 3 shows that it can earn at least $\Pi_B(q_n, 0.25q_n + 0.75)$ ($> \Pi_B(q^*, 0.25q^* + 0.75) > F$) by choosing $q_b = 0.25q_n + 0.75$. Then Lemma 3 shows $A$ can earn $\Pi_A(q_n, q^*_b(q_n)) - nF$ at most. Since these profits are smaller than $\Pi_A(Q, q^*_b(Q)) - F$, $A$ never chooses $n \geq 2$ products when $F < \Pi_B(q^*, 0.25q^* + 0.75)$.

Secondly, we consider the case that $\Pi_B(q^*, 0.25q^* + 0.75) \leq F < \Pi_B(q^*, Q)$. Define $\bar{q}_n = \{ q_n | \Pi_B(q_n, 0.25q_n + 0.75) = \Pi_B(q^*, Q) \}$. If $q_n \geq \bar{q}_n$, $B$ enters the low quality market because it can earn at least $\Pi_B(q_n, 0.25q_n + 0.75)$ ($> \Pi_B(q^*, Q) > F$) by choosing $q_b = 0.25q_n + 0.75$. Then $A$ can earn $\Pi_A(q_n, q^*_b(q_n)) - nF$ at most. If $q_k < q^*$, $B$ enters $q_b = Q$ and $A$ can earn $\max \{ \Pi_A(q_{k-1}, Q), \Pi_A(q_k, Q) \} - nF$ at most. If $\Pi_A(q_{k-1}, Q) > \Pi_A(q_k, Q)$, Proposition 1 shows that $B$ enters $q_b = Q$ because

$$\Pi_B(q_{k-1}, Q) > \Pi_B(q^*_b(Q), Q) = \Pi_A(q^*_b(Q))$$

$A$ earns $\Pi_A(q_{k-1}, Q) - nF$ in this case. Accordingly, all of $q_n < \bar{q}_n$, $q_k \geq q^*$, and $\Pi_A(q_{k-1}, Q) \leq \Pi_A(q_k, Q) (\leq \Pi_A(q^*, Q))$ must be satisfied in order to raise $A$’s net profit by deviation.

Define $\hat{q}_{k-1} = \{ q_{k-1} | \Pi_A(q_{k-1}, Q) = \Pi_A(q^*, Q) \}$. Then calculation shows that

$$\Pi_A(q, \phi) < \Pi_B(q^*, 0.25q^* + 0.75)(\leq F)$$

for any $q \leq \hat{q}_{k-1}$. Hence $A$ never chooses products smaller than $q^*_b(Q)$, and consequently $q_i \geq q^*$ must hold if $A$ chooses two or more products. However, numerical calculation shows that, when $q^* \leq q_1 < \cdots < q_n < \bar{q}_n$,

$$\max_{\Phi} \Pi_A(q_1, q_2, q_3; \phi) = \Pi_A(q^*, \sqrt[3]{\bar{q}_n}, \bar{q}_n; \phi)$$

$$< \Pi_A(q^*, \bar{q}_n, \phi) + \Pi_B(q^*, 0.25q^* + 0.75) \leq \Pi_A(q^*, \bar{q}_n, \phi) + F$$

and
\[
\Pi_A \left( \left( \hat{q}^*, \bar{q}_n \right), \phi \right) < \Pi_A \left( Q, \hat{q}_b^*(Q) \right) + \Pi_B \left( q^*, 0.25q^* + 0.75 \right) \\
\leq \Pi_A \left( Q, \hat{q}_b^*(Q) \right) + F.
\]

Therefore choosing two or more products decreases A’s net profit. Q.E.D.

Proof of Proposition 7.

If \( q_c < \min \{ q_A, q_b \} \), \( P_c = 0 \) in equilibrium (Note that only the products of the highest and the second highest quality have positive sales in equilibrium). Hence C chooses \( q_c \) such that \( q_b < q_c < q_A \). When \( q_c \leq 0.4q_A + 0.6q_b \),
\[
\Pi_c \left( q_A, q_b, q_c \right) = \frac{9(q_A - q_b)(q_A - q_c)(q_c - q_b)}{2q_c(4q_A - 3q_b - q_c)^2} \quad \text{and} \quad \frac{\partial \Pi_c}{\partial q_c} > 0.
\]

When \( q_c \geq 0.4q_A + 0.6q_b \),
\[
\Pi_c \left( q_A, q_b, q_c \right) = \frac{q_A - q_c}{18q_c} \quad \text{and} \quad \frac{\partial \Pi_c}{\partial q_c} < 0.
\]

Therefore \( \Pi_c \) is maximized at \( q_c = 0.4q_A + 0.6q_b \) for \( q_c \in [q_b, q_A] \). Entry is blockaded if \( \Pi_c \left( Q, \hat{q}_b^*(Q), 0.4q + 0.6q_b^*(Q) \right) \leq F \).

When \( F < \Pi_c \left( Q, \hat{q}_b^*(Q), 0.4q + 0.6q_b^*(Q) \right) \), B must choose closer product to A in order to deter entry of C. The reason is B cannot earn positive gross profit when C enters higher quality than B’s product. Define
\[
K = \frac{\partial}{\partial q_b} \Pi_A \left( q_A, q_B, \phi \right) / \frac{\partial}{\partial q_B} \Pi_c \left( q_A, q_B, 0.4q_A + 0.6q_B \right) \\
- \frac{\partial}{\partial q_A} \Pi_A \left( q_A, q_B, \phi \right) / \frac{\partial}{\partial q_A} \Pi_c \left( q_A, q_B, 0.4q_A + 0.6q_B \right).
\]

Then A continues choosing \( q_A = Q \) if \( K < 0 \) at \( q_A = Q \). Calculation shows that
\[
K = -\frac{3(3q_A - 2q_B - 1)(2q_A + 3q_B)^2}{10q_A^2q_B(q_A - q_B)} < 0 \quad \text{for} \quad q_b < 0.25q_a + 0.75
\]
and \( K = 0 \) for \( q_b \geq 0.25q_a + 0.75 \).

Thus A chooses \( q_A = Q \) and B chooses \( \bar{q}_B \) such that \( \Pi_c \left( Q, \bar{q}_b, 0.4q + 0.6\bar{q}_b \right) = F \) in the unique equilibrium for \( \Pi_c \left( Q, 0.25Q + 0.75, 0.55Q + 0.45 \right) \leq F \).
If \( F < \Pi_c(Q,0.25Q + 0.75,0.55Q + 0.45) \) and \( q_A = Q \), \( q_B \) must be larger than 0.25\( q_A \) + 0.75 to prevent entry of \( C \). Then \( K = 0 \) is satisfied and thus \( A \) is indifferent among choosing \( q_A \in [q_{sel}, Q] \) for \( F^* \leq F \). \( A \) chooses \( q_A \in [q_{wp}, Q] \) for \( F < F^* \), because choosing \( q_A < q_{wp} \) allows entry of \( B \) in \( q_B = Q \). \( B \) chooses \( q_B \) such that \( q_B < q_A \) and \( \Pi_c(q_A, q_B, 0.4q_A + 0.6q_B) = F \) to deter entry of \( C \) between \( A \) and \( B \). Q.E.D.

Proof of Proposition 8.
(a) A consumer of income \( y \) is willing to pay up to \( y(1 - q_i^{-1}) \) for good \( q_i \). When \( A \) chooses \( q_i \) and \( B \) does not enter, \( A \) offers \( P_1 = 1.5(1 - q_i^{-1}) \) in equilibrium and
\[
W = 0.5 \int_{1.5}^{3} y(1 - q_i^{-1}) dy - F
\]
\[
= \frac{27(q_i - 1)}{16q_i} - F.
\]
If \( F \geq \Pi_B(Q,q_B^*(Q),\phi) \), Proposition 4 shows \( q_i = Q \). Hence \( W = \frac{27(Q - 1)}{16Q} - F \).

(b) If \( \Pi_B(q^*, Q, \phi) \leq F < \Pi_B(Q, q_B^*(Q), \phi) \), \( A \) offers \( q_i \) that satisfies
\[
F = \frac{(q_i - 1)(5r + q_i r - 4q_i - 4)}{4(q_i^2 - r - 1)(q_i r - q_i - r - 1)} \quad \text{where} \quad r = \sqrt{q_i + 1}.
\]
Then
\[
W(q_i) = \frac{27(q_i - 1)}{16q_i} - \frac{(q_i - 1)(5r + q_i r - 4q_i - 4)}{4(q_i^2 - r - 1)(q_i r - q_i - r - 1)}
\]
and calculation shows that \( \frac{dW}{dq_i} > 0 \) and \( \frac{d^2W}{dq_i^2} > 0 \). Therefore \( \frac{dW}{dF} > 0 \) in this case.

(c) If \( \Pi_c(Q, q_B^*(Q), 0.4Q + 0.6q_B^*(Q)) \leq F < \Pi_B(q^*, Q, \phi) \), then \( (q_A, q_B) = (Q, q_B^*(Q)) \),
\[
P_A = \frac{3Q - 2q_B^*(Q) - 1}{2Q}, \quad \text{and} \quad P_B = \frac{q_B^*(Q) - 1}{q_B^*(Q)}
\]
in equilibrium. Consumers whose income are less than \( \frac{3Q - 4q_B^* + 1}{2Q - 2q_B} = b \) buy \( q_B \), and other consumers buy \( q_A \). Calculation
shows that \( b < 1.5 \). Since the compensating variations in our definition must be non-negative, \( 0.5 \int_0^1 y \left[ 1 - (q_b^*(Q))^{-1} \right] dy > \Pi_B (q, q_b^*(Q), \phi) \) \( (> F) \) must hold. Then

\[
W = 0.5 \int_0^1 y \left[ 1 - (q_b^*(Q))^{-1} \right] dy - F + 0.5 \int_{b}^{1.5} y (1 - Q^{-1}) dy + 0.5 \int_{1.5}^3 y (1 - Q^{-1}) dy - F
\]

\[
> 0.5 \int_b^{1.5} y (1 - Q^{-1}) dy + \frac{27(Q - 1)}{16Q} - F
\]

\[
> \frac{27(Q - 1)}{16Q} - F.
\]

Therefore \( W \) evaluated at \( F = \Pi_B (q^*, Q, \phi) - \varepsilon \) is larger than \( W \) at \( F = \Pi_B (q, q_b^*(Q), \phi) \).

(d) Since \( (Q_A, Q_B) = (Q, q_b^*(Q)) \) for \( \Pi_C (Q, q_b^*(Q), 0.4Q + 0.6q_b^*(Q)) \leq F < \Pi_B (q^*, Q, \phi) \),

\[
\frac{dW}{dF} = -2
\]

in this phase.

(e) If \( \Pi_C (Q, 0.25Q + 0.75, 0.55Q + 0.45) \leq F < \Pi_C (Q, q_b^*(Q), 0.4Q + 0.6q_b^*(Q)) \),

\( (q_A, q_B) = (Q, q_b) \) and

\[
W = 0.5 \int_1^c y (1 - q_b^{-1}) dy + 0.5 \int_c^3 y (1 - Q^{-1}) dy - 2F
\]

where \( c = \frac{3Q - 4q_b + 1}{2Q - 2q_b} \). Since \( \frac{\partial W}{\partial q_b} < 0 \) and calculation shows that \( \frac{\partial W}{\partial q_b} > 0 \),

\[
\frac{dW}{dF} < -2.
\]

(f) If \( F < \Pi_C (Q, 0.25Q + 0.75, 0.55Q + 0.45) \), \( q_B \geq 0.25Q + 0.75 \) in equilibrium. Then the proof of Lemma 1 shows that \( (p_A^*, p_B^*) = \left( \frac{5(q_A - q_B)}{3q_A}, \frac{q_A - q_B}{3q_B} \right) \) and \( y_{AB} = \frac{4}{3} \).

Therefore

\[
W = \frac{7}{36} (1 - q_b^{-1}) + \frac{65}{36} (1 - q_A^{-1}) - 2F.
\]

Note that \( \frac{\partial W}{\partial q_A} > 0 \) and \( \frac{\partial W}{\partial q_B} > 0 \). Proposition 7 shows that \( A \) chooses \( q_A \in \left[ q_{ub}, Q \right] \) if \( F^* \leq F < \Pi_C (Q, 0.25Q + 0.75, 0.55Q + 0.45) \) and \( q_A \in \left[ q_{ub}, Q \right] \) if \( F < F^* \). Therefore
\[ W_H = \{ W | q_A = Q \}, \quad W_L = \{ W | q_A = q_{bet} \} \quad \text{for} \quad F^* \leq F < \Pi_c(Q, 0.25Q + 0.75, 0.55Q + 0.45), \quad \text{and} \quad W_L = \{ W | q_A = q_{up} \} \quad \text{for} \quad F < F^*. \]

(f1) When \( F \) decreases and \( A \) chooses \( q_A = Q \), \( B \) must choose larger \( q_B \). Thus
\[
\frac{dW_H}{dF} < -2.
\]

(f2) When \( A \) chooses \( q_{bet} \) and \( B \) chooses \( q_B = 0.25Q + 0.75 \),
\[
W_L = \frac{7(Q - 1)}{36(Q + 3)} + \frac{65}{36} (1 - q_b^{-1}) - 2F
\]

where \( F = \Pi_c(q_{bet}, 0.25Q + 0.75, 0.4q_{bet} + 0.15Q + 0.45) = \frac{4q_{bet} - Q - 3}{48q_{bet} + 18Q + 54} \)

from the definition of \( q_{bet} \). Since \( \frac{dq_{bet}}{dF} > 0 \) and calculation shows that \( \frac{dW_L}{dq_{bet}} > 0 \),
\[
\frac{dW_L}{dF} > 0 \quad \text{for} \quad F^* \leq F < \Pi_c(Q, 0.25Q + 0.75, 0.55Q + 0.45).
\]

(f3) Since \( \frac{dq_{up}}{dF} < 0 \), \( B \) must choose larger \( q_B \) as \( F \) decreases when \( A \) chooses \( q_{up} \).

Consequently,
\[
\frac{dW_L}{dF} = \frac{\partial W_L}{\partial q_B} \frac{dq_B}{dF} + \frac{\partial W_L}{\partial q_{up}} \frac{dq_{up}}{dF} - 2 < -2 \quad \text{for} \quad F < F^*. \text{Q.E.D.}
\]
References


Martinez-Giralt, Xavier and Neven, Damien J. “Can Price Competition Dominate Market Segmentation?” Journal of Industrial Economics, June 1988, 36(4), pp. 431-


Figure 1: The optimal product of the first entrant
Figure 2: The optimal products in the extended model
Figure 3: The social welfare