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Study of the gauge mediation signal with nonpointing photons at the CERN LHC

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In this paper we study the gauge mediation signal with the ATLAS detector at the CERN Large Hadron Collider. We focus on the case where next lightest supersymmetric particle is the long-lived lightest neutralino ($\tilde{\chi}_1^0$), which decays predominantly into a photon ($\gamma$) and a gravitino ($\tilde{G}$). A nonpointing photon from the neutralino decay can be detected with good position and time resolution by the electromagnetic calorimeter (ECAL), while the photon momentum could be precisely measured if the photon were converted inside the inner tracking detector before reaching the ECAL. A new technique is developed to determine the masses of the slepton ($\tilde{\ell}$) and the neutralino from events with a lepton and a converted nonpointing photon arising from the cascade decay $\tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \rightarrow \ell \gamma \tilde{G}$. A Monte Carlo simulation at a sample point shows that the masses could be measured with an error of 3% for $O(100)$ selected $\ell \gamma$ pairs. Once the sparticle masses are determined by this method, the decay time and momentum of the neutralino are solved using the ECAL data and the lepton momentum only, for all $\ell \gamma$ pairs without the photon conversion. We estimate the sensitivity to the neutralino lifetime for $c\tau=10$ cm to $O(10)$ m.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) is a promising candidate for physics beyond the standard model. As supersymmetry (SUSY) must be spontaneously broken, the MSSM needs an additional sector (the hidden sector) which breaks the supersymmetry while avoiding the flavor-changing neutral current problem. The origin of SUSY breaking and the mediation to the MSSM sector are therefore the key feature of SUSY models. The hidden sector SUSY breaking is expressed in terms of the order parameter of the SUSY breaking $F$ and the scale of the SUSY breaking mediation to the MSSM sector $M$. The mass scale of the MSSM sparticles $M_{\text{SUSY}}$ is then of the order of $\lambda F/M$, where $\lambda$ is the coupling of the hidden sector to the MSSM sector. If $M \approx M_{\text{Pl}}$ (the Planck mass), $M_{\text{SUSY}}=1$ TeV corresponds to $\sqrt{\lambda} \approx 10^{10}$ GeV. This class of mediation is called the supergravity (SUGRA) model. On the other hand, the SUSY breaking mediation may be due to renormalizable interactions, such as the gauge interaction. This is called the “gauge mediation” (GM) model [1,2]. In the GM models $M$ and $F$ are arbitrary and we expect $M \ll M_{\text{Pl}}$.

The GM models are described by a few parameters. The MSSM gaugino masses $M_i$ ($i=1,2,3$) and slepton masses are of the order of $\alpha_i F/M$ in the simplest GM model, where $\alpha_i$ denotes each gauge coupling constant. On the other hand, the gravitino mass $m_{\tilde{G}}$ is proportional to $F_0/M_{\text{Pl}}$ where $F_0$ is the order of the SUSY breaking of the total system ($F_0 > F$) [3,4].

Because $M \ll M_{\text{Pl}}$, the lightest SUSY particle is the gravitino ($\tilde{G}$) in the GM models. The next lightest SUSY particle (NLSP) is a particle in the MSSM sector which decays into a gravitino [5]. If the lightest neutralino ($\tilde{\chi}_1^0$) is the NLSP, the dominant decay mode is $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}$. The neutralino lifetime is a function of $F_0$ and the neutralino mass $m_{\tilde{\chi}_1^0}$, and may be long lived.

The CERN Large Hadron Collider (LHC) is a $pp$ collider at a center of mass energy of 14 TeV. The LHC is expected to start its physics runs in 2007. The initial integrated luminosity will be $10$ fb$^{-1}$/yr at the beginning (low luminosity runs) and then upgraded to $100$ fb$^{-1}$/yr (high luminosity runs). Signatures of the GM model at the LHC are spectacular [6–9].

In the case of the neutralino NLSP, SUSY events nearly always have hard photons, which may not be pointing to the interaction point (nonpointing photons).

In this paper we propose a new approach to studying the signature of the GM model using the ATLAS detector at the LHC for the case where the neutralino NLSP predominantly decays into a photon and a gravitino with $c\tau$ longer than $O(10)$ cm. We use two newly developed techniques. One is to determine the direction of the gravitino momentum by using the arrival position, arrival time, and momentum of the nonpointing photon, which are measured at the electromagnetic calorimeter (ECAL). The precision of the photon momentum would be significantly improved if the nonpointing photon were converted into an $e^+ e^-$ pair in the inner tracking detector located inside the ECAL. The other technique is the “mass relation method,” a mass reconstruction technique which does not rely on the conventional end point measurement [10]. In this method we use the fact that each event from the same cascade decay satisfies mass shell conditions of intermediate particles. These techniques are described in Sec. II and a simulation is carried out in Sec. III. A fast Monte Carlo simulation shows that, using the new tech-
FIG. 1. Decay kinematics of the NLSP (the lightest neutralino $\tilde{\chi}_1^0$) in the ATLAS detector.

The statistical error of the masses of $\ell$ and $\tilde{\chi}_1^0$ can be a few percent for $O(100)$ selected $\ell \gamma$ pairs from the decay chain $\tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \rightarrow \ell \gamma \tilde{G}$. Although more work, especially full detector simulations, is needed to establish the techniques, the result of the fast simulation is quite encouraging.

In Sec. IV we show that events with $\ell \gamma$ pairs are fully reconstructed by using the measured mass and the ECAL information. This measurement is utilized to determine the neutralino lifetime for $10 \text{ cm} < c \tau < O(10) \text{ m}$. This analysis does not require the photon to be converted in the inner detector. Therefore the available number of events is significantly larger. Finally, in Sec. V we discuss how these measurements would be translated into the fundamental parameters $F_0$, $F$, and $M$ in the GM model.

II. KINEMATICS OF THE EVENTS WITH NONPROMPT $\tilde{\chi}_1^0$ DECAY

We first discuss the decay kinematics of the neutralino with non-negligible lifetime. In Fig. 1 we schematically show a neutralino decaying into a gravitino and a photon in the ATLAS detector. The neutralino is produced at the interaction point $O$ at the time $t=0$, and then flies to the decay point $D$, where the neutralino decays at $t=t_D$ into a photon and a gravitino. The photon goes to the point $A$ in the ECAL at $t=t_\gamma$. We define two angles $\alpha$ and $\psi$, the former between the photon momentum $p_\gamma$ and the position vector $\vec{x}_\gamma = \vec{OA}$, and the latter between $p_\gamma$ and the gravitino momentum $p_{\tilde{G}}$.

We can experimentally measure $\alpha$, $t_\gamma$, and the distance $L = |\vec{OA}|$. The angle $\psi$ can then be calculated from the three observables as

$$\cos \psi = \frac{1 - \xi^2}{1 + \xi^2},$$

where $\xi = \frac{ct_\gamma - L \cos \alpha}{L \sin \alpha}$. (1)

Because the three momenta $\vec{p}_\gamma$, $\vec{p}_\ell$, and $\vec{p}_{\tilde{G}}$ are on the same plane, the direction of the gravitino momentum is completely determined.\footnote{The formula can be easily shown by considering a Lorentz transformation into the frame where the interaction point and the photon detection point are identical. We define the rapidities of the gravitino and photon, taking the boost direction as the $z$ direction. To obtain the formula Eq. (1) one then goes back to the laboratory frame, noting the additivity of the rapidities. We thank Dr. Odagiri for pointing this out.}

The information of the gravitino direction may be used to determine the sparticle masses. We describe this idea for the cascade decay $\tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \rightarrow \ell \gamma \tilde{G}$. The sfermion $\tilde{\ell}$ may be copiously produced at the LHC from $\tilde{\chi}_1^0$ or $\tilde{\chi}_1^\pm$ decays, where $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ are predominantly produced from gluino or squark decays. The neutralino and sfermion masses ($m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\ell}}$) are related by the following formula:

$$m_{\tilde{\ell}}^2 = (p_\gamma + p_{\tilde{G}} + p_\ell)^2$$

$$= 2E_\ell E_{\tilde{G}}(1 - \cos \psi) + 2E_\ell E_\gamma(1 - \cos \theta_{\ell \gamma})$$

$$= \left(1 + \frac{E_\ell(1 - \cos \theta_{\ell \tilde{G}})}{E_\gamma(1 - \cos \psi)}\right)m_{\tilde{\chi}_1^0}^2$$

$$+ 2E_\ell E_\gamma(1 - \cos \theta_{\ell \gamma})$$

$$= am_{\tilde{\chi}_1^0}^2 + b,$$ (2)

where we use the relation

$$m_{\tilde{\chi}_1^0}^2 = (p_\gamma + p_{\tilde{G}})^2$$

$$= 2E_\ell E_{\tilde{G}}(1 - \cos \psi),$$ (3)

and neglect the lepton and gravitino masses. A pair of parameters ($a, b$) can be calculated event by event from the momenta of the lepton and photon and the direction of the gravitino momentum. Because the sparticle masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\ell}}$ should be common for all events, one can determine them if we have at least two tagged events.

In Refs. [10,11] the masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\ell}}$ are determined by measuring end points of mass distributions for events containing $\ell \ell \gamma$ which come from the decay chain $\tilde{\chi}_2^0 \rightarrow \ell \ell \tilde{\chi}_1^0 \rightarrow \ell \ell \gamma \tilde{G}$. End points in the distributions of invariant masses $m_{\ell \ell}$, $m_{\ell \gamma}$, and $m_{\ell \ell \gamma}$ are combined to solve $m_{\tilde{\ell}}$, $m_{\tilde{\chi}_1^0}$, and $m_{\tilde{\chi}_2^0}$. Note that only the events near the end points contribute to the mass determination in the end point analysis.

Our proposal is quite different from the end point analysis. We assume that a set of events come from a common cascade decay, and use the mass shell conditions of the particles involved in the cascade decay. Each event gives an independent constraint to the masses as given in Eq. (2) and contributes to the mass determination. We call this “the mass relation method.” This technique may be applied for other
cascade decays of SUSY particles, which will be discussed in future publications.

The ATLAS detector at the LHC has a good capability to measure nonpointing photons, where the barrel part of the electromagnetic calorimeter and the transition radiation tracker (TRT) will play important roles (Fig. 1). The barrel ECAL is a liquid-argon calorimeter covering a pseudorapidity range $|\eta|<1.4$. The inner radius of the ECAL is 150 cm. We assume the angular resolutions of the photon arrival point at the ECAL inner surface to be $\sigma_{\eta}=0.004$ and $\sigma_{\phi}=0.002$. The ECAL energy resolution is expected to be $\sigma_{E_{\gamma}}/E_{\gamma}=10%/\sqrt{E_{\gamma}}$, where the photon energy $E_{\gamma}$ is given in GeV. The longitudinal and transverse segmentation of the ECAL gives a measure of the development of electromagnetic showers. The first longitudinal sampling is finely segmented in the $\eta$ direction, resulting in a good angular resolution $^3$ of $\sigma_{\eta}=60$ mrad/\sqrt{E_{\gamma}} , where $\theta$ is the polar angle of the photon momentum with respect to the beam axis. The azimuthal angle $\phi$ of the photon momentum is only poorly measured by the ECAL, as the segmentation is very coarse in this direction. The ECAL also has an excellent time resolution, $\sigma_{\tau_{\gamma}}<100$ ps for $E_{\gamma}>30$ GeV, confirmed by a test-beam experiment.

When a photon is pointing to the interaction point, the photon momentum is precisely determined by the ECAL only, namely, by measuring the energy deposit and the arrival position. However, in the case of the GM model, the photon is in general nonpointing, and the transverse components of the photon momentum are only poorly measured. Fortunately, the barrel TRT is located inside the barrel ECAL as a component of the inner tracking detector. This detector covers the radial range from 56 to 107 cm and the pseudorapidity range $|\eta|<0.7$. As the straw tube trackers of the barrel TRT are parallel to the beam axis, trajectories of charged particles are precisely measured in the $r-\phi$ plane. If a photon is converted into an $e^+e^-$ pair before escaping the barrel TRT, the $\phi$ angle of the photon momentum can be very precisely measured. As the $\phi$ angle resolution is much better than the $\theta$ angle resolution by the ECAL, the resolution of the angle $\alpha$ becomes $\sigma_{\alpha}=\sqrt{\sigma_{\phi}^2+\sigma_{\theta}^2}-\sigma_{\theta}$. The material thickness of the TRT is about 10% of one radiation length at $\eta\sim 0$; namely, $\sim 10\%$ of photons will be converted in the TRT.

### III. RECONSTRUCTION OF GRAVITINO DIRECTION AND SPARTICLE MASSES

In order to test the new techniques we perform a Monte Carlo event simulation at GM point G1 [10,11] with $F_{0}\nu$, or equivalently the neutralino lifetime $c\tau$, being the only free parameter. The model parameters and some of the sparticle masses are listed in Table I. The low energy SUSY parameters are calculated by ISASUSY in ISAJET 7.51 [12], and the mass spectrum, the couplings, and the decay branching ratios are interfaced to HERWIG 6.4 [13]. We generate $10^5$ SUSY events at this point, corresponding to an integrated luminosity of 13.9 fb$^{-1}$. When we simulate the events, we keep the lightest neutralinos (NLSPs) stable at the generator level. Then a fast detector simulator ATLAST is used for all particles except the neutralinos. The decay of the neutralinos and the photon conversions are simulated at the analysis stage. The photon conversion probability is estimated based on the detector thickness of the TRT [10]. If (a) a neutralino decays into a photon and a gravitino before escaping the TRT region, (b) the photon points to the barrel ECAL, and (c) the photon is converted inside the barrel TRT, then the energy, position, time, and direction of the photon are smeared by the Gaussian distribution according to their resolutions. The detector resolutions assumed in the simulation are listed in Table II. We assume that the time resolution of the ECAL is constant, $\sigma_{\tau_{\gamma}}=100$ ps, for $E_{\gamma}>30$ GeV.

For a moment we set the neutralino lifetime to be $c\tau=100$ cm. In this case the neutralinos efficiently decay in the TRT, as the outer radius of the TRT is roughly 100 cm. We first apply preselection cuts to suppress the standard model background:

\begin{equation}
\begin{aligned}
& (i) \quad M_{\text{eff}} > 400 \text{ GeV}, \\
& (ii) \quad E_T^{\text{miss}} > 0.1 M_{\text{eff}}.
\end{aligned}
\end{equation}

The missing transverse energy $E_T^{\text{miss}}$ is calculated from the reconstructed jets, leptons, photons, and unreconstructed particles.

### Table I. Model parameters and some of the sparticle masses at point G1. The parameter $N$ is an integer number which appears in Eq. (10).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sparticle masses (GeV)</th>
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<tr>
<td>$F/M=90$ TeV</td>
<td>$m_{\tilde{g}}=720$</td>
</tr>
<tr>
<td>$M=500$ TeV</td>
<td>$m_{\tilde{q}_i}=958$</td>
</tr>
<tr>
<td>$N=1$</td>
<td>$m_{\tilde{g}}=915$</td>
</tr>
<tr>
<td>$\tan\beta=5$</td>
<td>$m_{\tilde{h}_{12}}=909$</td>
</tr>
<tr>
<td>$\mu&gt;0$</td>
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### Table II. Detector resolutions assumed in our Monte Carlo simulation of nonpointing photons. The photon energy $E_{\gamma}$ is given in GeV. As for the $\phi$ angle of the photon momentum, we assume photon conversion in the TRT detector.

<table>
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<th>Observable</th>
<th>Detector</th>
<th>Resolution</th>
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<td>Photon energy</td>
<td>$E_{\gamma}$</td>
<td>ECAL</td>
</tr>
<tr>
<td>Photon arrival time</td>
<td>$t_{\gamma}$</td>
<td>ECAL</td>
</tr>
<tr>
<td>Photon arrival position</td>
<td>$\eta$</td>
<td>ECAL</td>
</tr>
<tr>
<td>$\phi$</td>
<td>ECAL</td>
<td>0.004</td>
</tr>
<tr>
<td>Photon momentum</td>
<td>$\theta$</td>
<td>ECAL</td>
</tr>
<tr>
<td>$\phi$</td>
<td>TRT</td>
<td>0.001</td>
</tr>
</tbody>
</table>

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$^3$The good angular resolution is obtained for photons originating from the interaction point. As the ECAL has a pointing tower structure, the angular resolution for nonpointing photons, which is still under study, might be degraded.
calorimeter energies. The effective mass \( M_{\text{eff}} \) is defined by the sum of the missing transverse energy and the transverse momenta of the four hardest jets:

\[
M_{\text{eff}} = E_T^{\text{miss}} + p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4}.
\]

We do not include photons from the neutralino decays in the above calculation. The efficiency of the preselection cuts for the generated SUSY events is 80%.

The following cuts are then applied to select good "non-pointing" photons with conversion in the TRT:

1. \( E_\gamma > 30 \text{ GeV} \),
2. \( \alpha > 0.2 \),
3. \( \Delta t_\gamma = t_\gamma - L/c > 1.0 \text{ ns} \).

The distributions of \( E_\gamma \), \( \alpha \), and \( \Delta t_\gamma \) are shown in Figs. 2(a)–2(c), where the cuts (1)–(3) are sequentially applied.

In Fig. 2(d) we plot \( \Delta \psi = \psi - \psi_{\text{true}} \), where \( \psi \) is calculated from the measured \( L \), \( \alpha \), and \( t_\gamma \) using Eq. (1) and \( \psi_{\text{true}} \) is the true value obtained from the generator information. The resolution \( \sigma_\psi \) is better than 40 mrad in this case.

In order to determine the masses of the slepton and the neutralino which appear in the cascade decay \( \tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0 \rightarrow \ell \gamma\tilde{\chi}_1^0 \), isolated leptons (electrons and muons) with transverse momentum larger than 20 GeV are searched for to make a pair with the nonpointing photon. If there are several leptons in an event, we choose the \( \ell \gamma \) pair which minimizes the invariant mass \( m_{\ell \gamma} \). The parameters \( a \) and \( b \) are calculated for each \( \ell \gamma \) pair. The scatter plot in the \((a,b)\) plane is shown in Fig. 3(a), where the sample contains about 120 \( \ell \gamma \) pairs. The points are clearly concentrated along a line. The section of the \( b \) axis and the slope of the line must correspond to \( m_{\tilde{\chi}_1^0}^2 \) and \( m_{\tilde{\ell}}^2 \), because of the relation \( b = m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2 a \). In Fig. 3(b) we take a simple average of \((a,b)\) for the events between the two dotted lines in Fig. 3(a) by dividing the region of \( a \) into nine bins, and we fit the averaged data by the linear function Eq. (2). The fit results are \( m_{\tilde{\ell}} = 162.1 \text{ GeV} \) and \( m_{\tilde{\chi}_1^0} = 117.5 \text{ GeV} \), while the input values are 161.7 GeV and 117.0 GeV, respectively.

To estimate the errors of the mass measurement, the simulation and the fit are repeated 100 times with different random number seeds. The fitted masses are plotted in Fig. 4. By fitting the distribution, we obtain the errors of the masses as \( \sigma_{m_{\tilde{\ell}}} = 2.7 \text{ GeV} \) and \( \sigma_{m_{\tilde{\chi}_1^0}} = 3.5 \text{ GeV} \). They correspond to relative mass errors of \( 2\sim3\% \). If one of the sparticle masses is precisely determined by some other measurements, the other sparticle mass can be determined with a precision of \( \sim 300 \text{ MeV} \) by the mass relation method.

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\( ^4 \)Isolation from tracks/clusters is yet to be examined.
IV. FULL RECONSTRUCTION AND LIFETIME MEASUREMENT

In this section we demonstrate full reconstruction of the cascade decay \( \tilde{\ell} \rightarrow \tilde{\chi}^{0}_1 \rightarrow \ell \gamma \tilde{\chi} \), and we show that the neutralino lifetime can be determined by the reconstructed decay time and momentum of the neutralino. This analysis becomes possible after the precise determination of \( m_{\tilde{\ell}} \) and \( m_{\tilde{\chi}^0_1} \) described in the previous section.

Here we study events with leptons and nonpointing photons, where the photons may or may not be converted in the inner detector. For each \( \ell \gamma \) pair, the arrival time \( t_\gamma \), arrival point \( x_\gamma \), and energy \( E_\gamma \) of the photon, the longitudinal component of the photon momentum, and the lepton momentum \( \vec{p}_\ell \) would be directly measured, while the transverse components of the photon momentum, the neutralino four-momentum \( \vec{p}_{\tilde{\chi}} \), and its decay time \( t_D \) \( (< t_\gamma) \) are not directly measured. On the other hand, we have the following equations involving the unknown quantities,

\[
\begin{align*}
\vec{u}_{\tilde{\chi}} t_D + \vec{u}_\gamma (t_\gamma - t_D) &= \vec{x}_\gamma, \\
(p_{\tilde{\chi}} + p_\gamma)^2 &= p_{\tilde{\chi}}^2 = m^2_{\tilde{\chi}}, \\
(p_{\tilde{\chi}} - p_\gamma)^2 &= p_G^2 = m^2_G = 0, \\
p_{\tilde{\chi}}^2 &= m^2_{\chi^0_1},
\end{align*}
\]

(7)

where \( \vec{u}_{\tilde{\chi}} \) and \( \vec{u}_\gamma \) are velocity vectors \( (\vec{u}_\gamma = c) \), and the lepton and gravitino masses are neglected. Provided that we know the two masses \( m_{\tilde{\ell}} \) and \( m_{\chi^0_1} \), we can solve for all the unknown parameters from the equations. There are two possible solutions for each \( \ell \gamma \) pair, which we obtain by numerically solving Eq. (7).

The simulation and analysis are modified from those of the previous section. The neutralino must decay inside the ECAL and the photon from the neutralino decay must enter the barrel ECAL, resulting in a much larger fiducial decay volume. We assume the fiducial volume as \( r < 150 \text{ cm} \) and \( |z| < 300 \text{ cm} \), where \( r \) and \( z \) are the radial distance from the beam axis and the distance from the interaction point along the beam axis, respectively. In addition, as the photon conversion is not required in the inner detector, the acceptance of the events further increases. After applying the preselection cuts given in Eq. (4), we select nonpointing photons by the following cuts:

1. \( E_\gamma > 30 \text{ GeV} \),
2. \( \Delta \theta > 0.2 \text{ rad} \),
3. \( \Delta t_\gamma > 1 \text{ ns} \).

Here \( \Delta \theta = \theta_\gamma - \theta_{\tilde{\ell}} \) is the difference between the polar angles of \( \vec{p}_\gamma \) and \( \vec{x}_\gamma \), where the polar angles are measured from the beam axis. Note that the cut (2) is different from that in Eq. (6), because one cannot determine the angle \( \alpha \) without precise measurement of the \( \phi \) angle using the photon conversion. This \( \Delta \theta \) cut is less efficient than the \( \alpha \) cut in Eq. (6). However, the overall selection efficiency of this analysis is much larger because we have a larger fiducial decay volume and do not require photon conversion in the inner detector.

Isolated leptons with transverse momentum larger than 20 GeV are used to make \( \ell \gamma \) pairs. For each \( \ell \gamma \) pair, there are two solutions for the decay kinematics. We take solutions if the reconstructed decay points are in the fiducial volume (e.g., \( r < 100 \text{ cm} \) and \( |z| < 300 \text{ cm} \)). Therefore each \( \ell \gamma \) pair may be counted twice. If there are several leptons in an event, all leptons are tried for each nonpointing photon and accepted if the \( \ell \gamma \) pair satisfies the fiducial condition.

In Fig. 5(a) we show a scatter plot of the reconstructed decay time \( t_D \) and the true decay time for \( c\tau = 100 \text{ cm} \). Here we use the input masses \( m_{\tilde{\ell}} \) and \( m_{\chi^0_1} \) for reconstruction. This plot shows that the neutralino decay time is correctly reconstructed for a significant fraction of the events. We also show the distribution of \( t_D / \gamma_\chi \) in Fig. 5(b) for \( c\tau = 50 \text{ cm} \), 100 cm, and 200 cm, where the factor \( 1/\gamma_\chi \) corrects the effect of Lorenz boost. The distribution shows the expected exponential damping toward large \( t_D / \gamma_\chi \) values. The geometric acceptance of the events would be small when \( c\tau > R_m \) or \( \tau \gamma_\chi < 1 \text{ ns} \), where \( R_m (= 150 \text{ cm}) \) is the inner radius of the ECAL and 1 ns is our cut value on the difference of the photon arrival times.

The geometric effect on the acceptance may be corrected by a study with full detector simulations. The momentum distribution of the neutralino would be an important uncertainty, as the transverse momentum distribution should depend on the gluino and squark masses. The heavier sparticle masses may be measured precisely by the \( j\tilde{\chi}_1^0 \) or \( j\tilde{\chi}_1^0 \) invariant mass distribution for selected jets \( (j) \) and a neutralino, where the neutralino momentum is reconstructed by solving Eqs. (7). Therefore, we assume that the systematics due to the momentum distribution of the neutralino would be small enough.

Assuming that the systematic errors are controlled, we may study the sensitivity to the lifetime \( c\tau \) using the two measured values (a) \( \langle t_D / \gamma_\chi \rangle \), the average of the corrected decay time; and (b) \( N_{\ell\gamma} \), the number of accepted \( \ell \gamma \) pairs (note that we do not count an \( \ell \gamma \) pair more than once to avoid overcounting). They are plotted as functions of \( c\tau \) in

![Graph showing the distribution of reconstructed decay time vs true decay time for different values of cτ.]  

FIG. 5. (a) The distribution of the neutralino decay time \( t_D \) vs the true value. (b) Distributions of \( t_D / \gamma_\chi \) for \( c\tau = 50 \text{ cm} \) (dashed), 100 cm (solid), and 200 cm (dotted).
This function is numerically obtained by fitting the average values in the region within 10 cm for cτ, where cτ takes its maximum at 1 ns. Note that we do not have the sensitivity on ψ when cτ<10 cm because the time resolution is ~0.1 ns, and c×0.1 ns ~3 cm. The number of ℓγ pairs with photon conversion (N^{conv}_{ℓγ}) is 13.0 and 80.5 for cτ =10 cm and cτ=30 cm, respectively. As the number of converted nonpointing photons is rather small for cτ =10 cm, a large integrated luminosity is needed to determine the sparticle masses. For large cτ values, the average ⟨t_D/γ⟩ is saturated since most of the neutralinos decay outside the detector. Indeed, the number of reconstructed events takes its maximum at cτ ~100 cm and decreases monotonically as shown in Fig. 6(b).

The sensitivity of the measurement to the lifetime cτ is estimated by the error of the measurement Δ⟨t_D/γ⟩; namely we define the error of the lifetime Δcτ by the following formula:

\[ g(cτ±Δcτ) = ⟨t_D/γ⟩ ± Δ⟨t_D/γ⟩ , \]

where g(cτ) is a function to describe the average ⟨t_D/γ⟩. This function is numerically obtained by fitting the ⟨t_D/γ⟩ to a second power polynomial function of cτ using the average values in the region within cτ±50 cm. When cτ >100 cm, most of the neutralinos decay after reaching the ECAL and ⟨t_D/γ⟩ loses sensitivity to cτ. On the other hand, the number of ℓγ pairs N_{ℓγ} is noticeably reduced when cτ is longer than 100 cm. We can estimate the sensitivity of N_{ℓγ} to cτ in a same way. The result is shown in Fig. 7 and summarized in Table III. Note that we ignore the errors of m_ℓ and m_γ for the moment and use the input mass values. Note also that only statistical errors are considered here. Systematic errors are yet to be studied with full detector simulations.

For large cτ values the number of selected events is limited. In Table IV we summarize N_{ℓγ} and N^{conv}_{ℓγ} for an integrated luminosity of 13.9 fb^{-1}. For cτ<2000 cm the number N^{conv}_{ℓγ} still exceeds 130 events if we assume an integrated luminosity of 100 fb^{-1} at point G1. Therefore, it might still be possible to determine the neutralino and slepton masses.

Table III. Relative resolution of cτ. As in Figs. 7(a) and 7(b), the values are estimated from the average ⟨t_D/γ⟩ and the number of ℓγ pairs N_{ℓγ}, respectively.

<table>
<thead>
<tr>
<th>cτ (cm)</th>
<th>Δcτ/cτ</th>
<th>cτ (cm)</th>
<th>Δcτ/cτ</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.173</td>
<td>200</td>
<td>0.062</td>
</tr>
<tr>
<td>20</td>
<td>0.062</td>
<td>250</td>
<td>0.038</td>
</tr>
<tr>
<td>30</td>
<td>0.041</td>
<td>300</td>
<td>0.034</td>
</tr>
<tr>
<td>40</td>
<td>0.036</td>
<td>350</td>
<td>0.036</td>
</tr>
<tr>
<td>50</td>
<td>0.037</td>
<td>400</td>
<td>0.030</td>
</tr>
<tr>
<td>60</td>
<td>0.027</td>
<td>450</td>
<td>0.031</td>
</tr>
<tr>
<td>70</td>
<td>0.037</td>
<td>500</td>
<td>0.040</td>
</tr>
<tr>
<td>80</td>
<td>0.039</td>
<td>600</td>
<td>0.025</td>
</tr>
<tr>
<td>90</td>
<td>0.043</td>
<td>700</td>
<td>0.037</td>
</tr>
<tr>
<td>100</td>
<td>0.045</td>
<td>800</td>
<td>0.037</td>
</tr>
<tr>
<td>150</td>
<td>0.043</td>
<td>900</td>
<td>0.037</td>
</tr>
<tr>
<td>200</td>
<td>0.047</td>
<td>1000</td>
<td>0.039</td>
</tr>
<tr>
<td>250</td>
<td>0.061</td>
<td>1500</td>
<td>0.048</td>
</tr>
<tr>
<td>300</td>
<td>0.070</td>
<td>2000</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Table IV. Number of ℓγ pairs for an integrated luminosity of 13.9 fb^{-1}. N^{conv}_{ℓγ} is the number of ℓγ pairs with photon conversion in the inner detector.

<table>
<thead>
<tr>
<th>cτ (cm)</th>
<th>N_{ℓγ}</th>
<th>N^{conv}_{ℓγ}</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>147</td>
<td>13.0</td>
</tr>
<tr>
<td>30</td>
<td>1424</td>
<td>80.5</td>
</tr>
<tr>
<td>100</td>
<td>3193</td>
<td>116.3</td>
</tr>
<tr>
<td>300</td>
<td>2413</td>
<td>72.8</td>
</tr>
<tr>
<td>2000</td>
<td>536</td>
<td>14.3</td>
</tr>
</tbody>
</table>

V. Determination of the Fundamental Parameters

In this section we utilize the measurement of the sparticle masses and the neutralino lifetime to determine more funda-
mental parameters in the GM scenario. From the measurement of the slepton and neutralino masses, the ratio $F/M$ is determined with a precision of a few percent. This can be seen in the expressions for the masses at the messenger scale as [14]

$$M_i = \frac{\alpha_i(M)}{4\pi} \frac{F}{M} N \times g(F/M^2),$$

$$m_i^2 = \frac{3\alpha_i(M)}{40\pi^2} \frac{F^2}{M^2} N \times f(F/M^2),$$

$$m_{\tilde{t}}^2 = \left( \frac{3\alpha_i(M)}{160\pi^2} + \frac{3\alpha_i^2(M)}{32\pi^2} \right) \frac{F^2}{M^2} N \times f(F/M^2),$$

(10)

where $N$ is an integer number, while $g$ and $f$ are some functions that satisfy $f(0) = g(0) = 1$ for $F \ll M^2$. The sparticle masses are proportional to $F/M$ in this limit.

The absolute size of $M$ (or $F$) is rather difficult to determine because it appears only through the sfermion mass running from $M$ to the SUSY scale. A study [10] shows that the relative error $\Delta M/M$ is $\sim 30\%$ with 1% mass errors at this point.

The neutralino lifetime depends on the order parameter of the total SUSY breaking $F_0$ and the neutralino mass $m_{\tilde{\chi}}^0$ [5];

$$c\tau = \frac{1}{k_\gamma} \left( \frac{100}{m_{\tilde{\chi}}^0} \right)^5 \left( \frac{\sqrt{F_0}/100}{\text{TeV}} \right)^4 \times 10^{-2} \text{ cm},$$

(11)

where $k_\gamma = |N_{11} \cos \theta_w + N_{12} \sin \theta_w|^2$ with $\theta_w$ being the Weinberg angle, and $N_{ij}$ is the neutralino mixing angles. The constant is $k_\gamma = \cos^2 \theta_w$ for the Bino-like $\tilde{\chi}_1^0$. The parameter $F_0$ is also related to the gravitino mass itself [3,4];

$$m_{\tilde{G}} = \frac{1}{\sqrt{\sqrt{F_0}/100}} M_{pl} \approx 2.4 \text{ eV}.$$

(12)

The sensitivity to the neutralino lifetime is about 4% for $c\tau \sim 100$ cm as given in the previous section. Ignoring the errors of the neutralino mass and $k_\gamma$, this is translated to an error of about 1% for $\sqrt{F_0}$. The lifetime $c\tau = 100$ cm corresponds to $\sqrt{F_0} \sim 1000$ TeV and $m_{\tilde{G}} \sim 0.2$ keV, respectively.

If $c\tau$ is $\mathcal{O}(10) \sim \mathcal{O}(100)$ m, the ratio of the number of $\ell\gamma$ pairs to that of SUGRA-like events is a more sensitive measure for the lifetime. For $c\tau = 2000$ cm ($\sqrt{F_0} \sim 2400$ TeV and $m_{\tilde{G}} \sim 1.4$ keV), the estimated error is about 4%. This corresponds to a 1 (2) % error on $\sqrt{F_0} (m_{\tilde{G}})$. The mass error of $\tilde{\chi}_1^0$ is estimated to be around 3%, because the number of events with a converted photon is around 100 for an integrated luminosity of 100 fb$^{-1}$, which is obtained in one year at the high luminosity runs of the LHC. From Eqs. (10) and (12) the error of $F_0 (m_{\tilde{G}})$ due to the mass error is 3 (6) % in this case. For $c\tau = 10^3$ cm the number of events with a lepton and a converted nonpointing photon is 70 for the ultimate integrated luminosity of 300 fb$^{-1}$.

The above estimate is rather optimistic because we do not consider systematic errors of the measurement. In addition, we have so far not considered the effect of background events from the standard model processes and the SUSY production itself, where prompt photons from the interaction point mimic nonpointing photons due to the limited detector resolution. Note that, in our Monte Carlo sample, 28 757 sleptons are produced, and $\sim 2500$ ($\sim 70$) of them are accepted as the sample with a (converted) nonpointing photon for $c\tau = 300$ cm. Especially for a large $c\tau$ case, the number of real nonpointing photons is small, and the analysis might severely suffer from the background. Studies with full detector simulations are necessary to understand the systematic errors and the background and should be completed before the start of the LHC physics run. If events with a nonpointing photon converting in the TRT cannot be used for mass reconstruction due to the backgrounds, $\Delta m_{\tilde{\chi}_1^0} \sim 30\%$ is expected from the end point analysis involving jets and leptons. In this case the uncertainty of the neutralino mass would lead to an error of the gravitino mass by a factor of 2.

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