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The effect of risk aversion on distribution channel contracts: implications for return policies

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Abstract

The return policy is often used in retailing supply chains. However, it is controversial in judging their practical value. In the literature, various theoretical and modeling explanations of why the return policy is used in practice and is preferred by the retailer and manufacturer have been put forth. The literature focusing on the channel agents' risk attitudes to explain the adoption seems to have led to the conclusion that the two agents' preferences for adopting the full-return policy over the no-return policy are always in conflict, and thus the risk attitudes do not explain the adoption of return policy in practice. In this paper, we reinvestigate this issue. We first identify two distinct phases of risk averseness, high or low, for each of the two agents. We show distinct behaviors of how the wholesale price and order size are set in each phase. Then, we show that the full-return policy can be preferred over the no-return policy by both the agents if both of them are high risk averse. This implies that the agents' risk attitudes can explain the adoption of return policy. This is a new theoretical result, which is contrary to the existing understanding in the literature. Our result highlights the importance and intricacy of channel policies especially when the risk attitudes of agents are considered.

Keywords: Supply chain contract; Risk aversion; Return policies

1. Introduction

Return policies are often used in retailing supply chains. However, it is controversial when their practical value is assessed carefully (Padmanabhan and Png, 1995). Retailers may procure excessive inventory because they can return its unsold inventory without penalty. In spite of such a possibility, an incentive may be necessary for risk-averse retailers to procure the sufficient inventory that leads to superior channel coordination. Because of its managerial importance and inherent complexity, return policies have been studied in the literature, especially from the modeling and theoretical perspective (Pasternak, 1985; Marvel and Peck, 1995; Kandel 1996; Padmanabhan and Png, 1997; Emmons and Gilbert, 1998; Lau and Lau, 1999; Donohue, 2000; Webster and Weng, 2000; Tsay, 2001; Tsay, 2002).

The risk attitudes should be critical in designing the channel contracts. In general, the more risk-averse the retailer is, the less inventory is expected to be procured. Facing a
more risk-averse retailer than the usual ones, should the manufacturer set the wholesale price lower to induce a larger order, or set it higher to directly increase the revenue? If the manufacturer by itself is more risk-averse than the usual ones, then how does this affect the consequence? We further address the following question: How does the interaction of two agents’ risk attitudes affect the design and adoption of return policy?

Of the extensive literature on supply chain coordination in general, few studies address the effect of risk attitudes upon the manufacturer and retailer relationship (See Tsay, 1999; Cachon, 2003). Of the modeling literature on the return policy, Tsay (2002) is an exceptional and influential paper that addresses the effect of risk attitudes of channel agents upon the contracts involving the return policy. He concludes that the two agents’ preferences for adopting the full-return policy over the no-return policy are always in conflict, and thus the risk attitudes of agents do not explain why the return policy is used in practice.

We use in this paper the same model as that in Tsay (2002). The risk-averse retailer faces the one-time demand that linearly decreases in retail price and takes either a low or high level of demand. The risk-averse manufacturer sets a wholesale price, and then the retailer determines its order quantity. After the demand is realized, the retailer determines the retail price to maximize its profit. The risk attitudes are represented by the mean-standard deviation (MS) value function. The MS value adjusts the mean downward by its standard deviation times a nonnegative risk sensitivity parameter. The retailer and manufacturer in the model maximize their respective MS values instead of their expected profits prior to the demand realization. All the information including the demand and the two agents’ risk attitudes is shared by them. The manufacturing cost is assumed to be zero for simplicity of arguments.

In this paper, we identify two phases of risk averseness, low and high, for both the retailer and manufacturer. The wholesale price and order size for the no-return policy change with respect to the retailer’s risk sensitivity. We show that they change discontinuously as the risk averseness shifts from the low phase to the high phase. The same can be shown with respect to manufacturer’s risk sensitivity. This identification of two risk averseness phases, high or low, leads to the following propositions. First, the manufacturer prefers the full-return policy to the no-return policy if and only if the retailer’s risk sensitivity is greater than the manufacturer’s risk sensitivity. This is consistent with the common expectation that the manufacturer takes advantage of the lower risk averseness of the two. Second, we show that the high risk-averse manufacturer tends to set the wholesale price low to avoid the high profit variance situation associated with the full-return policy. This setting of low wholesale price is required for the retailer to prefer the full-return policy. Together, if the manufacturer is high risk-averse and the retailer has even higher risk-averseness, then both the agents can prefer the full-return policy over the no-return policy. The contribution of this paper to the modeling literature investigating why the return policy is used in practice is that the agents’ risk attitudes can explain the adoption of return policy. This is a new result, which is contrary to Tsay (2002)’s result described above.

The rest of this paper is organized as follows. In Section 2, we review the extensive literature using modeling approaches to justify the use of return policy over the no-return policy. In Section 3, we formalize the model used in this paper. In Section 4, we derive the two types of equilibrium outcomes for each of the no-return and full-return policies, which result for the two phases of risk averseness for each of the two parties. In Section 5, we discuss how the wholesale price and order size change in the different phases of risk averseness, and investigate which of the two policies is preferred by the two channel agents. In Section 6, we discuss the differences between
Tsay (2002) and this paper. In Section 7, we summarize the insights derived in this paper.

2. Literature Review

In this section, we review the literature using modeling approaches to justify the use of return policy over the no-return policy. In Section 2.1, we review the papers that assume risk neutral channel agents. These papers typically justify the return policy as a scheme to coordinate the supply chain in the sense of optimizing the expected performance of entire supply chain. Some papers justify the return policy as a scheme to achieve Pareto-improvement in the sense that all the agents involved are no worth off, and at least one agent is strictly better off using it over the no-return policy. In Section 2.2, we review the literature that explains the adoption of the return policy from the perspective of agents’ risk attitudes. In this context, the coordination in the sense of optimizing a single objective function for the entire supply chain is difficult to formalize because the relationship between individual risk averseness and the integrative objection function cannot be articulated. Therefore, the Pareto-improvement is used to justify the return policy. Please refer to Gan et al. (2004) and Chiu and Choi (2013) for a variety of definitions of coordination in supply chain. The influential paper of Tsay (2002) shows that the channel agents’ preference of the return policy over the no-return policy is always conflicting when the agents’ risk attitudes are considered, and thus agents’ risk attitudes do not explain why the return policy is used in practice. This somewhat negative result has led the researchers to explore other reasons to justify the use of return policy, which is reviewed in Section 2.3. In this paper, on the contrary to Tsay (2002), we show that risk sensitivity alone in such a basic supply chain setting explains why the full-return policy can be preferred by both the manufacturer and retailer.

2.1. Return policy with risk-neutral agents

Cachon (2003) and Tsay et al. (1999) provide comprehensive reviews of contract models in supply chain. Cachon (2003) provides an extensive review covering various contract types, multiple period or location models, asymmetric information models, and so on. He mainly considers the contract models to coordinate a supply chain. Tsay et al. (1999) provide a classification scheme for the literature on contracts in the supply chain management context. The classification scheme is based on the specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buyback or return policies, allocation rules, lead times, and quality.

The no-return policy is called a wholesale price contract in the supply chain coordination literature. Lariviere and Porteus (2001) analyze the no-return policy in detail. This simple contract is commonly observed in practice as the standard way to govern transactions in supply chain. At the same time it is known as a contract that does not maximize the supply chain wide expected profit. Double marginalization causes the inefficiency in supply chain (Spengler 1950).

Assuming that both the retailer and manufacturer are risk-neutral, the return policy has been investigated as a scheme to coordinate the supply chain while regarding the no-return policy as its basic benchmark transaction policy. Pesternack (1985) is early work analyzing a return policy using the newsvendor model. He shows that channel coordination can be achieved under a return policy where the retailer returns either all unsold inventories for a partial credit, or a certain portion of his original order for a full credit. Emmons and Gilbert (1998) extend the work of Pesternack (1985) by incorporating retailer’s pricing decision and shows that both the retailer and
manufacturer can increase their expected profit using a return policy under certain conditions. Bernstein and Federgruen (2005) prove that a buyback contract cannot optimize the supply chain wide expected profit in price-dependent demand with uncertainty if the wholesale and buyback prices are constant. Chen and Bell (2011) investigate a return policy under the situation that the retailer is experiencing customer returns and price-dependent demand with uncertainty. They prove that a buyback contract cannot optimize the supply chain wide expected profit in price-dependent demand with uncertainty if the wholesale and buyback prices are constant. Chen and Bell (2011) investigate a return policy under the situation that the retailer is experiencing customer returns and price-dependent demand with uncertainty. They prove that a buyback contract cannot optimize the supply chain in the setting. They propose a buyback scheme that includes two buyback prices, one for unsold inventory and the other for customer returns to achieve supply chain optimization. For further discussions on the variations of return policy or buyback contract with customer returns, the reader is referred to Ruiz-Benítez and Muriel (2014) and the references therein.

Bose and Anand (2007) investigate a return policy in price-independent demand with uncertainty, considering the models in which the wholesale price is exogenously fixed. They show that when the wholesale price is sufficiently high, the equilibrium return policy achieves Pareto-improvement over a no-return policy (a price-only contract). That is, the supply chain members are no worse off with the return policy in place than with the no-return policy. However, they show that, in general, the equilibrium return policy does not achieve Pareto-improvement.

Some papers consider the return policy using the price-dependent additive demand model in which demand is modeled as the sum of a price-sensitive deterministic function and a non-price-sensitive positive random variable (Padmanabhan and Png, 1997; Yao et al., 2008; Gurnani et al., 2010; Zhao et al., 2014). The model considered in this paper follows this category of model. Zhao et al. (2014) extend Padmanabhan and Png (1997) considering the full-return with partial or full credit and both the supplier’s and retailer’s perspectives in the supply chain. They show that, depending on the demand uncertainty level, Pareto-improvement can be achieved using the buyback contract, and the demand uncertainty level can be a critical factor for affecting the applicability of supply chain contract.

There are other papers investigating the return policy in the price-dependent multiplicative demand model (Emmons and Gilbert, 1998; Bernstein and Federgruen, 2005; Granot and Yin, 2005; Song et al., 2008). In the model, demand is modeled as the product of a price-sensitive deterministic function and a non-price-sensitive positive random variable. As they point out in the literature, the major results derived in the analysis are valid only for the buyback contract in the price-dependent multiplicative demand model.

2.2. Return policy with risk-averse agents

Few studies address the effect of risk attitudes of channel agents upon the selection of contracts involving return policies. Lau and Lau (1999) and Choi et al. (2008) study return policies considering the interaction between the risk-averse retailer and manufacturer. Lau and Lau (1999) assume that both the agents have mean-variance objective functions and investigate through numerical experiments how their risk averseness affects the pricing and return-credit strategy for the manufacturer. They demonstrate that the risk-averse manufacturer sets both the wholesale price and return-credit lower than the risk-neutral one, and the manufacturer facing the risk-neutral retailer decreases the return-credit almost to zero. Choi et al. (2008) carry out a mean-variance analysis of the supply chain consisting of the risk-averse retailer and manufacturer under the return policy. They consider the entire supply chain’s mean-variance optimization problem and show how a return policy can be used for supply chain coordination. In the centralized supply chain, they show that a changing
return-credit implies a reallocation of profit and risk between the two parties.

Tsay (2002) extends the model of Padmanabhan and Png (1997) incorporating the retailer's and manufacturer's risk sensitivities. He concludes that the preferences of the manufacturer and retailer for the full-return policy are always conflicting with each other. Ohmura and Matsuo (2012) study the wholesale price contract with a risk-neutral manufacturer and a risk-averse retailer, and show the effects of the retailer's risk aversion on the supply chain performance under the no-return policy. Ohmura (2014) also studies the wholesale price contract with a risk-neutral manufacturer and a risk-averse retailer and numerically shows that the dynamics of interaction is different depending on the risk measurement used in the analysis such as the mean-variance and the conditional value-at-risk.

There are several approaches to model a risk-averse newsvendor (See Choi et al., 2011). If we represent the risk attitude of the newsvendor by the utility function of von Neumann and Morgenstern (1944), then the risk-averse newsvendor maximizes its expected utility function. Another well-known approach for representing the risk attitude is the mean-risk approach. The mean-variance function categorized in this approach is used in the context of portfolio optimization (Markowitz, 1959). A trade-off between the mean outcome and its variance is assessed using this function as the risk measure. It is also known that the utility function can be approximated by the mean-variance function if it is quadratic or if it is normally distributed. Thus, the mean-variance function is often used in finance and economics. The newsvendor model using the mean-variance function is discussed in Anvari (1987) and Chen and Federgruen (2000). Chiu and Choi (2013) review the literature focusing on mean-variance analytical models. Recently, some studies use the Conditional Value-at-Risk (CVaR) to model the risk-averse newsvendor (Chen et al., 2009; Wu et al., 2014; Dai and Meng, 2015; Xu et al., 2015). Choi et al. (2011) show that CVaR actually represents a trade-off between the expected profit and a certain risk measure, and thus can be regarded as a special mean-risk criterion. The mean-standard deviation (MS) value function that we consider in this paper is also a mean-risk approach. It uses the standard deviation instead of the variance as its risk measurement. Although it is not much used in the literature, Bar-Shira and Finkelstain (1999) argue that the use of MS value function is more robust than the approaches based on expected utility. This is because the value of MS function increases in mean and decreases in standard deviation. The MS value function is also mathematically tractable. Since the dimensional units of mean and standard deviation are the same, Tsay (2002) points out that this representation intuitively "dollarizes" an agent's aversion to risk.

2.3. Other justifications for return policy

Recent papers have explored alternative explanations for the preference of the return policy over the no-return policy. In the context of new product introduction under a monopolistic and channel leading retailer, Matsui (2010) shows that the retailer and consumers share the same interest regarding a new product introduction under the full-return policy. In this case, the full-return policy removes the retailer's excessive risk averseness by shifting it to the manufacturer. Yue and Raghunathan (2006) show that the information asymmetry plays a role for increasing manufacturer's preference of the full-return policy. They discuss the case where the retailer has more information on demand than the manufacturer. In the context of supply-chain-to-supply-chain competition, Ai et al. (2012) derive the conditions under which the preferences of both the retailers and manufacturers for the channel policy are consistent. They assume that
both the agents are risk neutral. He and Zhao (2012) investigate coordination in a three-echelon supply chain under supply and demand uncertainty. They propose a return policy used by the manufacturer and retailer, along with the no-return policy used by the raw-material supplier and the manufacturer. The combined policy can coordinate the supply chain. Bandyopadhyay and Paul (2010) consider two capacity-constrained manufacturers competing for the shelf space of the same retailer. They show that the full-return policy is the only possible equilibrium of the game in such an environment. That is, the manufacturers offer the full-return policy because of the effect of the shelf competition, rather than the ordinal sense of supply chain coordination. These papers suggest various explanations of why the full-return policy is used in practice.

In this paper, we reinvestigate the effect of the risk sensitivity of manufacturer and retailer upon their inventory policy to justify the use of full-return policy under the basic supply chain model discussed in Section 2.2. On the contrary to Tsay (2002)’s result, the full-return policy over no-return policy can be preferred by two high risk-averse channel agents. This explanation of why the full-return policy is used in practice is new in the literature, and it implies the importance of considering the risk attitudes of channel agents in designing channel contracts.

3. The model and notation

In this paper, we use the same model as that in Tsay (2002); however, we focus on the case in which the manufacturer is the Stackelberg leader. We assume throughout the paper that the manufacturing cost $c$ is zero for simplicity of arguments, as in Tsay (2002) when he analyzes the equilibrium. Table 1 summarizes the notation used in this paper.

The model discussed in this paper involves a manufacturer and a retailer dealing with a single product over a single period. The decision sequence protocol is shown in Table 2 and described as follows. The retailer is risk-averse, and faces the demand $q$ that is equal to a random primary demand $\alpha$ minus the slope $-\beta$ multiplied by the retail price $p$. The primary demand takes on a low value $\alpha_l$ with probability $\lambda$ and a high value $\alpha_h$ with probability $1 - \lambda$.

The model assumes symmetric information. That is, the information on the demand curve $(\beta, \alpha_l, \alpha_h, \lambda)$ and the risk sensitivity (MS value functions with $k_R$ and $k_M$) is shared by the two parties at the beginning. The risk-averse manufacturer as the Stackelberg leader sets the unit wholesale price $w$, and then the risk-averse retailer determines its order quantity $s$. After the primary demand is realized, the retailer determines the retail price to maximize its profit. The retailer can return the overstock with full credit for the full-return policy case.

The risk attitude is represented by the mean-standard deviation (MS) function. The MS value factors in risk attitude by adjusting the mean downward by the amount equal to its standard deviation times a nonnegative risk sensitivity parameter $k$. The retailer and manufacturer in the model maximize their respective MS values instead of their expected profits prior to the demand realization.

Tsay (2002) introduced the risk-adjusted probability, $\Lambda = \lambda + k\sqrt{\lambda (1 - \lambda)}$ for $0 \leq k < \sqrt{(1 - \lambda)/\lambda}$ (i.e., $\lambda \leq \Lambda < 1$). In this paper as in Tsay (2002), we consider $k$ that satisfies $0 \leq k < \sqrt{(1 - \lambda)/\lambda}$. By Lemma 1 of Tsay (2002), MS values can be expressed as a linear combination of the profits for the two demand scenarios. To verify this, let $z$ be a random profit that takes on a value $\pi_l$ with probability $\lambda$ and a value $\pi_h$ with probability $1 - \lambda$. Then, the MS value function with respect to $z$ can be transformed as in (1). Therefore, $\Lambda$ is interpreted as the risk-adjusted probability of low
demand scenario occurring.

\[
MS(z) = E(z) - k\sqrt{Var(z)} = (\lambda \pi_l + (1 - \lambda)\pi_h) - k\sqrt{\lambda(1 - \lambda)(\pi_h - \pi_l)^2} \\
= \left(\lambda + k\sqrt{\lambda(1 - \lambda)}\right)\pi_l + \left[1 - \left(\lambda + k\sqrt{\lambda(1 - \lambda)}\right)\right]\pi_h = \lambda \pi_l + (1 - \lambda)\pi_h
\]

\[\tag{1}\]

**Table 1 - The notations of the model**

<table>
<thead>
<tr>
<th>i</th>
<th>index for demand scenario; (i \in {l, h}) for low and high primary demand, respectively.</th>
</tr>
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<tbody>
<tr>
<td>(\alpha_i)</td>
<td>primary demand; (\alpha_l) with probability (\lambda) and (\alpha_h) with probability (1 - \lambda), where (\alpha_h &gt; \alpha_l &gt; 0) and (0 &lt; \lambda &lt; 1);</td>
</tr>
<tr>
<td>(\beta)</td>
<td>market sensitivity to retail price, where (\beta &gt; 0);</td>
</tr>
<tr>
<td>(p_i)</td>
<td>retail price in demand scenario (i);</td>
</tr>
<tr>
<td>(q_i)</td>
<td>market demand in demand scenario (i) for; (q_i \equiv \alpha_i - \beta p_i);</td>
</tr>
<tr>
<td>(s)</td>
<td>retailer’s order size;</td>
</tr>
<tr>
<td>(w)</td>
<td>unit wholesale price charged by the manufacturer;</td>
</tr>
<tr>
<td>(c)</td>
<td>manufacturer’s unit production cost;</td>
</tr>
<tr>
<td>(\Pi^R)</td>
<td>retailer’s profit;</td>
</tr>
<tr>
<td>(\Pi^M)</td>
<td>manufacturer’s profit;</td>
</tr>
<tr>
<td>(\pi^R_i)</td>
<td>realization of (\Pi^R) in demand scenario (i);</td>
</tr>
<tr>
<td>(\pi^M_i)</td>
<td>realization of (\Pi^M) in demand scenario (i);</td>
</tr>
<tr>
<td>(k_R)</td>
<td>retailer’s risk sensitivity, where (0 \leq k_R &lt; \sqrt{(1 - \lambda)/\lambda});</td>
</tr>
<tr>
<td>(k_M)</td>
<td>manufacturer’s risk sensitivity, where (0 \leq k_M &lt; \sqrt{(1 - \lambda)/\lambda});</td>
</tr>
<tr>
<td>(MS^R)</td>
<td>retailer’s MS value function; (MS^R \equiv E(\Pi^R) - k_R * \text{StdDev}(\Pi^R));</td>
</tr>
<tr>
<td>(MS^M)</td>
<td>manufacturer’s MS value function; (MS^M \equiv E(\Pi^M) - k_M * \text{StdDev}(\Pi^M));</td>
</tr>
<tr>
<td>(A_R)</td>
<td>risk-adjusted probability, where (A_R = \lambda + k_R\sqrt{\lambda(1 - \lambda)}) and (\lambda \leq A_R &lt; 1).</td>
</tr>
<tr>
<td>(A_M)</td>
<td>risk-adjusted probability, where (A_M = \lambda + k_M\sqrt{\lambda(1 - \lambda)}) and (\lambda \leq A_M &lt; 1).</td>
</tr>
</tbody>
</table>

**Table 2 - Decision sequence protocol**

| Stage 0: | The manufacturer and retailer share the information on the demand curve \((\beta, \alpha_l, \alpha_h, \lambda)\) and the risk sensitivity (MS value functions with \(k_R\) and \(k_M\)). |
| Stage 1: | The manufacturer declares distribution policy and determines the unit wholesale price \(w\). |
| Stage 2: | The retailer determines the order size \(s\). |
| Stage 3: | A primary demand, \(\alpha_l\) or \(\alpha_h\), is realized. |
| Stage 4: | The retailer determines the retail price \(p_i\) and returns the overstock if so allowed. |

4. **Equilibrium**

In this section, the method of backward induction is used to obtain the equilibrium of the game defined in the previous section. For the no-return policy case and full-return policy case, we solve the maximization problems faced by the two agents sequentially and backwardly in the decision sequence protocol in Table 2. As it becomes clear, the manufacturer’s risk sensitivity does not matter for the non-return policy case, and the retailer’s risk sensitivity does not matter for the full-return policy case.

4.1. **No-return policy**

In Stage 4 for the case of no-return policy, given the non-refundable on-hand inventory \(s\), the retailer determines the retail price \(p\) for the realized primary demand \(\alpha_h\) or \(\alpha_l\) that maximizes the ex post revenue.
Maximize \( p_i \pi_i = (\alpha_i - \beta p_i)p_i - ws \) subject to \( \alpha_i - \beta p_i \leq s \) for \( i = 1 \) and \( h \). \hspace{1cm} (2)

To solve this optimization problem, we need to consider two cases, \( \alpha_i/2 \leq s \leq \alpha_h/2 \) and \( s \leq \alpha_i/2 \) as in Figure 1. The case of \( s > \alpha_h/2 \) can be shown dominated by the case of \( s = \alpha_h/2 \), and thus we eliminate this case from our discussion below. In the case of \( \alpha_i/2 \leq s \leq \alpha_h/2 \), for the realization of primary demand \( \alpha_i \), we can easily verify in Figure 1a that the revenue of \( p = \alpha_i/2\beta \) times \( q = \alpha_i/2 \) is the maximum, and thus the retailer chooses the optimal price \( p = \alpha_i/2\beta \). Likewise, for the primal demand \( \alpha_h \), the optimal revenue is \( p = (\alpha_h - s)/\beta \) times \( q = s \). In the case of \( s \leq \alpha_i/2 \) as in Figure 1b, given the on-hand inventory \( s \), the optimal price is \( (\alpha_i - s)/\beta \) and \( (\alpha_h - s)/\beta \), respectively for \( \alpha_i \) and \( \alpha_h \).

\[
\begin{align*}
\text{Figure 1a} & - \alpha_i/2 \leq s \leq \alpha_h/2 \\
\text{Figure 1b} & - s \leq \alpha_i/2
\end{align*}
\]

\( \text{Figure 1} \) - The retailer's optimal pricing decisions for the cases of no-return policy

In Stage 2, given the unit wholesale price \( w \), the retailer determines its order size \( s \) so that it maximizes its MS value function. Based on (1) and the above derivation for Stage 4, the MS value function can be represented as follows:

\[
\begin{align*}
MS_R(s) = \begin{cases} \\
\Lambda R \frac{\alpha_i^2}{4\beta} + (1 - \Lambda R) \left( \frac{\alpha_h - s}{\beta} \right) - ws & \text{for } \alpha_i/2 \leq s \leq \alpha_h/2 \\
\Lambda R \left( \frac{\alpha_i - s}{\beta} - s \right) + (1 - \Lambda R) \left( \frac{\alpha_h - s}{\beta} \right) - ws & \text{for } s \leq \alpha_i/2 \\
\end{cases}
\end{align*}
\hspace{1cm} (3)
\]

where \( \Lambda R = \lambda + k_R \sqrt{\lambda(1 - \lambda)} \) and \( 0 \leq k_R < \sqrt{(1 - \lambda)/\lambda} \).

The retailer's optimal order size \( s \) can be derived by maximizing (3) with respect to \( w \) as follows. We eliminate the description for the case of \( w > \bar{\alpha}_R/\beta \), which is dominated by the case of \( \bar{\alpha}_R/\beta = w \).

\[
\begin{align*}
s^* = \begin{cases} \\
\frac{1}{2} \left( \frac{\beta w}{1 - \Lambda R} \right) & \text{if } 0 < w \leq \frac{(1 - \Lambda R)(\alpha_h - \alpha_i)}{\beta} \\
\bar{\alpha}_R - \beta w & \text{if } \frac{(1 - \Lambda R)(\alpha_h - \alpha_i)}{\beta} \leq w \leq \frac{\bar{\alpha}_R}{\beta} \\
\end{cases}
\end{align*}
\hspace{1cm} (4)
\]

where \( \bar{\alpha}_R = \Lambda R \alpha_i + (1 - \Lambda R) \alpha_h \).

In Stage 1, anticipating the retailer's order size \( s^* \) in (4), the manufacturer determines the unit wholesale price \( w \) that maximizes its profit function, \( \Pi^M(w) = ws^* \). That is,
4.2. Full-return policy

In Stage 4, for the realized primary demand, the retailer determines the retail price \( p \) subject to \( q \geq 0 \) so that it maximizes its own profit. Here, since the retailer can return its excess inventory without any penalty cost, we assume that it has a sufficient amount of inventory. The optimization problem can be formalized as follows:

\[
\text{Maximize}_{p_i} \pi_i^{R} = (p_i - w)q_i \quad \text{subject to} \quad q_i \geq 0 \text{ for } i = l \text{ and } h.
\]  

To solve this optimization problem, we need to consider two cases, \( 0 \leq w \leq \frac{\alpha_l}{\beta} \) and \( \frac{\alpha_l}{\beta} \leq w \leq \frac{\alpha_h}{\beta} \) as in Figure 2. The case of \( w > \frac{\alpha_h}{\beta} \) can be shown dominated by the case of \( w = \frac{\alpha_h}{\beta} \), and thus we eliminate this case from our discussion. In the case of \( \frac{\alpha_l}{\beta} \leq w \leq \frac{\alpha_h}{\beta} \) as in Figure 2b, if \( \alpha_l \) is realized, then \( q \) is set at 0 because the wholesale price is greater than or equal to any possible retail price. For \( \alpha_h \), the profit is unit profit \( (p - w) \) times the corresponding demand \( q_h = \alpha_h - \beta p \), which is maximized at \( (\alpha_h + \beta w)/2\beta \). In this case, the retailer’s order size is \( q_h = \alpha_h - \beta p = (\alpha_h - \beta w)/2 \), and the entire order is returned if \( \alpha_l \) is realized. In the case of \( 0 \leq w \leq \frac{\alpha_l}{\beta} \), we can verify similarly in Figure 2a that the optimal price is \( (\alpha_l + \beta w)/2\beta \) for \( i = l \) and \( h \), respectively.

![Figure 2a - 0 ≤ w ≤ α_l/β](image)

![Figure 2b - α_l/β ≤ w ≤ α_h/β](image)

Figure 2 - The retailer’s optimal pricing decisions for the cases of full-return policy

In Stage 2, given the unit wholesale price \( w \), the retailer determines its order size \( s \). The retailer does not benefit from having inventory greater than \( q_h \). Therefore, the retailer will choose \( s = q_h = (\alpha_h - \beta w)/2 \) so as to have on hand exactly the optimal inventory for the high primary demand.

In Stage 1, anticipating the retailer’s order size \( s \), the manufacturer determines the unit wholesale price \( w \) that maximizes its MS value function \( MS^M = \Lambda_M q_l w + (1 - \Lambda_M)q_h w \). Based on the discussion for Stage 4, the manufacturer’s MS value is represented as follows.
\[ MS^M(w) = \begin{cases} 
\Lambda M \frac{\alpha_l - \beta w}{2} + (1 - \Lambda M) \frac{\alpha_h - \beta w}{2} & \text{if } 0 \leq w \leq \frac{\alpha_l}{\beta} \\
(1 - \Lambda M) \frac{\alpha_h - \beta w}{2} & \text{if } \frac{\alpha_l}{\beta} \leq w \leq \frac{\alpha_h}{\beta} 
\end{cases} \]

where \( \overline{\alpha}_M = \Lambda M \alpha_l + (1 - \Lambda M) \alpha_h \).

### 4.3. Two equilibrium outcomes for each of the no-return policy and full-return policy

In this section, we discuss the equilibrium outcomes for both the no-return and full-return cases. Let us define \( \Lambda' = 1 - (\alpha_l/(\alpha_h - \alpha_l))^2 \). We first discuss the case of \( \lambda \leq \Lambda' \), and then discuss the case of \( \lambda > \Lambda' \) noting how the earlier arguments change.

Figure 3 shows the regions of \( \lambda \leq \Lambda' \) and \( \lambda > \Lambda' \). As shown in Figure 3, the region \( \lambda > \Lambda' \) consists of two sub-regions \( \Lambda' \leq 0 < \lambda \) and \( 0 < \Lambda' < \lambda \). The sub-region \( \Lambda' \leq 0 < \lambda \) implies that \( \alpha_h \leq 2\alpha_l \), which means that the difference between the high and low demand is relatively low. In the sub-region \( 0 < \Lambda' < \lambda \), although the difference between the high and low demand is relatively high, the probability of experiencing the high demand \( 1 - \lambda \), is low. That is, \( \lambda > \Lambda' \) is the case where the market uncertainty is relatively low while \( \lambda \leq \Lambda' \) is the case where the market uncertainty is relatively high.

The consideration of risk sensitivity conceivably becomes critical in the case of \( \lambda \leq \Lambda' \).

We shall derive the equilibrium outcomes for the full-return policy case by assuming \( \lambda \leq \Lambda' \). Equations (7) can be rewritten as follows:

\[ MS^M(w) = \begin{cases} 
-\beta \left( w - \overline{\alpha}_M \right)^2 + \frac{\overline{\alpha}_M^2}{8\beta} & \text{if } 0 \leq w \leq \frac{\alpha_l}{\beta} \\
-\frac{(1 - \Lambda M)\beta}{2} \left( w - \frac{\alpha_h}{2\beta} \right)^2 + \frac{(1 - \Lambda M)\alpha_h^2}{8\beta} & \text{if } \frac{\alpha_l}{\beta} \leq w \leq \frac{\alpha_h}{\beta} 
\end{cases} \]

As described in the Appendix, we can show that the optimal wholesale price \( w^* = \alpha_h/2\beta \) and \( w^* = \overline{\alpha}_M/2\beta \) for \( \Lambda M \leq \Lambda' \) and \( \Lambda M \geq \Lambda' \), respectively.

For the no-return policy case, the manufacturer’s profit function in (5) can be written using (4) as follows:
\[ \Pi^M(w) = \begin{cases} 
-\frac{\beta}{2(1-\Lambda_R)} \left( w - \frac{(1-\Lambda_R)\alpha_h}{2\beta} \right)^2 & \text{if } 0 \leq w \leq \frac{(1-\Lambda_R)(\alpha_h - \alpha_l)}{\beta} \\
-\frac{\beta}{2} \left( w - \frac{\alpha_h}{2\beta} \right)^2 + \frac{(1-\Lambda_R)\alpha_h^2}{8\beta} & \text{if } (1-\Lambda_R)(\alpha_h - \alpha_l) \leq w \leq \frac{\alpha_h}{\beta} \end{cases} \]  

(9)

It can be shown that the optimal wholesale price is \( w = (1-\Lambda_R)\alpha_h/2\beta \) and \( w = \bar{w}_R/2\beta \) for \( \Lambda_R \leq \Lambda' \) and \( \Lambda_R \geq \Lambda' \), respectively. The reader is referred to Ohmura and Matsuo (2012) for the detailed derivation of optimal wholesale price.

Note that there exist two types of equilibrium outcomes for each of the no-return and full-return cases. As the risk sensitivity \( k_M (k_R) \) increases, \( \Lambda_M (\Lambda_R) \) increases proportionately. Therefore, as the risk sensitivity increases, one type of equilibrium outcomes shifts to the other. In that sense, we call the manufacturer (retailer) with \( \Lambda_M \leq \Lambda' (\Lambda_R \leq \Lambda') \) as "Low risk-averse manufacturer (retailer)" and the manufacturer (retailer) with \( \Lambda_M \geq \Lambda' (\Lambda_R \geq \Lambda') \) as "High risk-averse manufacturer (retailer)".

Based on the discussions so far, it is straightforward to represent the equilibrium outcomes as in Table 3.

**Table 3 - Equilibrium outcomes for the no-return policy and full-return policy when \( \lambda \leq \Lambda' \)**

(Note: \( \bar{\alpha}_M = \Lambda_M \alpha_l + (1-\Lambda_M)\alpha_h \), \( \bar{\alpha}_R = \Lambda_R \alpha_l + (1-\Lambda_R)\alpha_h \), and \( \Lambda' = 1 - \left( \frac{\alpha_l}{\alpha_h - \alpha_l} \right)^2 \))

<table>
<thead>
<tr>
<th>No-return Policy</th>
<th>Full-return Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low risk-averse</td>
<td>High risk-averse</td>
</tr>
<tr>
<td>( \Lambda_R \leq \Lambda' )</td>
<td>( \Lambda_R \leq \Lambda' )</td>
</tr>
<tr>
<td>( w^* ) ( \frac{(1-\Lambda_R)\alpha_h}{2\beta} )</td>
<td>( \bar{\alpha}_R ) ( \frac{\alpha_h}{2\beta} )</td>
</tr>
<tr>
<td>( s^* ) ( \frac{\alpha_h}{4} )</td>
<td>( \bar{\alpha}_R ) ( \frac{\alpha_h}{4} )</td>
</tr>
<tr>
<td>( MS^{M*} ) ( \frac{(1-\Lambda_R)\alpha_h^2}{8\beta} )</td>
<td>( \bar{\alpha}_R ) ( \frac{\alpha_h^2}{8\beta} )</td>
</tr>
<tr>
<td>( MS^{R*} ) ( \frac{1}{4\beta} \left( \Lambda_R \alpha_h^2 + (1-\Lambda_R)\alpha_h^2 \right) )</td>
<td>( \frac{\alpha_h^2}{16\beta} )</td>
</tr>
</tbody>
</table>

Now let us assume \( \lambda > \Lambda' \) instead of \( \lambda \leq \Lambda' \). As shown in the Appendix, we can show that there exists only one type of equilibrium outcomes for each distribution policy. The equilibrium for each of the no-return policy and full-return policy corresponds to that for the high risk-averse agent (i.e., \( \Lambda' \leq \Lambda_R \) or \( \Lambda' \leq \Lambda_M \)) discussed above. Hence, for \( \lambda > \Lambda' \), the forms of equilibrium outcomes do not shift as the risk sensitivities \( k_M \) and \( k_R \) change.

4.4. The effects of the increase of risk sensitivity upon equilibrium outcomes

In this section, we assume \( \lambda \leq \Lambda' \) in order to fully describe the effect of risk sensitivity.

Figures 4 and 5 depict how the equilibrium outcomes change as the risk sensitivity of each agent increases, using the parameter values of \( \alpha_h = 27, \alpha_l = 10, \beta = 1, \) and \( \lambda = 0.3 \). Note that \( \Lambda' = 1 - (\alpha_l/(\alpha_h - \alpha_i))^2 = 0.654 \), and \( k' \) is determined by \( \Lambda' = \lambda + k' \sqrt{\lambda(1-\lambda)} \) as \( k' = 0.772 \). Figure 4 corresponds to the case of no-return policy where the manufacturer is not exposed to any risk, and the wholesale price \( w \) and the order size \( s \) depend on the retailer’s risk sensitivity. Figure 5 corresponds to the case of full-return policy where the manufacturer’s risk sensitivity determines the \( w \) and \( s \).
We shall discuss the two cases separately in detail.

4.5. Phase transition for the no-return policy:
For the no-return policy. Figure 4a is the case of $5 = \alpha_l/2 \leq s \leq \alpha_h/2 = 13.5$, and this case corresponds to the phase of retailer’s low risk averseness. Figure 4b is the case of $s \leq \alpha_l/2 = 5$, and this case corresponds to the phase of retailer’s high risk averseness. The left and right shaded areas represent the retailer’s negative and positive profits, respectively.

The left hand side in Figures 4c to 4f corresponds to the phase of retailer’s low risk averseness, and the right hand side corresponds to the phase of retailer’s high risk averseness. In the low phase, as the retailer’s risk sensitivity $k_R$ increases, the manufacturer reduces the wholesale price $w$ (as in Figure 4c), but the retailer keeps a constant order size $s$ (as in Figure 4d). At the threshold value of $k_R = k' = 0.772$, the manufacturer increases the wholesale price discontinuously, and the retailer reduces the order size in response. These discontinuous changes are results from the phase transition corresponding to the shift from Figure 4a to Figure 4b. With its high risk averseness, the retailer resorts to the setting of $s$ less than $\alpha_l/2$, which leads to a lower variation of profit or loss in Figure 4b than that in Figure 4a. As $k_R$ increases after passing the threshold value, both the $w$ and $s$ decrease.

At $k_R = k'$, the values of the first and second equations in (9) become equal. Therefore, the manufacturer’s MS value, or equivalently the profit, in Figure 4f is continuous at $k_R = k'$. However, the retailer’s MS value decreases discontinuously as in Figure 4e.

4.6. Phase transition for the full-return policy:
For the full-return policy. Figure 5a is the case of $10 = \alpha_l/\beta \leq w \leq \alpha_h/\beta = 27$, and this case corresponds to the phase of manufacturer’s low risk averseness. Figure 5b is the case of $0 \leq w \leq \alpha_l/\beta = 10$, and this case corresponds to the phase of manufacturer’s high risk averseness. The shaded areas represent the retailer’s profits.

The left hand side in Figures 5c to 5f corresponds to the phase of manufacturer’s low risk averseness, and the right hand side corresponds to the phase of manufacturer’s high risk averseness. In the low phase, the manufacturer keeps a constant wholesale price $w$, and the retailer in response also keeps a constant order size $s$. At the threshold value of $k_M = k' = 0.772$, the manufacture decreases the wholesale price discontinuously, and the retailer increases the order size in response. These discontinuous changes are results from the phase transition corresponding to the shift from Figure 5a to Figure 5b. The manufacturer’s MS value in Figure 5f is continuous at $k_M = k'$ by the same reason as that for the above no-return policy case. With the manufacturer’s high risk averseness, the manufacturer reduces $w$ considerably to make the positive profit for each of the demand scenarios as in Figure 5b. With the low setting of $w$, the retailer can set $s$ high to actually increase the profit especially when the retailer’s risk sensitivity is low. This can be seen in Figure 5e.
Figure 4a - Demand function and order size for no-return policy with the low risk-averse retailer, $k_R = k_R^L (= 0.327)$.

Figure 4b - Demand function and order size for no-return policy with the high risk-averse retailer, $k_R = k_R^H (= 0.905)$.

Figure 4c - Wholesale price w.r.t. the retailer’s risk sensitivity for no-return policy.

Figure 4d - Order size w.r.t. the retailer’s risk sensitivity for no-return policy.

Figure 4e - The retailer’s MS value w.r.t. the retailer’s risk sensitivity for no-return policy.

Figure 4f - The manufacturer’s MS value w.r.t. the retailer’s risk sensitivity for no-return policy.

Figure 4 - Phase transition for the no-return policy using the following parameters ($\alpha_r=27$, $\alpha_l=10$, $\beta=1$, $\lambda=0.3$).
Figure 5a - Demand function and order size for full-return policy with the low risk-averse manufacturer, $k_M = k_M^L (= 0.4.)$

Figure 5b - Demand function and order size for full-return policy with the high risk-averse manufacturer, $k_M = k_M^H (= 1.271.)$

Figure 5c - Wholesale price w.r.t. the manufacturer’s risk sensitivity for full-return policy

Figure 5d - Order size w.r.t. the manufacturer’s risk sensitivity for full-return policy

Figure 5e - The retailer’s MS value w.r.t. the manufacturer’s risk sensitivity for full-return policy

Figure 5f - The manufacturer’s MS value w.r.t. the manufacturer’s risk sensitivity for full-return policy

Figure 5 - Phase transition for the full-return policy using the following parameters ($\alpha_H = 27$, $\alpha_l = 10$, $\beta = 1$, $\lambda = 0.3$).
5. The two parties’ preferences for the full-return policy

In this section, we evaluate the adoption of full-return policy vs. no-return policy. Depending on the combination of a high or low risk-averse manufacturer and a high or low risk-averse retailer, the effect of changing the distribution policy from the no-return policy to the full-return policy is different. Taking the differences between the two appropriate values in Table 3, we can derive Table 4 as shown in the Appendix. Here, \( \bar{A}_R \) is a value such that \( \Delta(MS^R) = 0 \) when \( A_R \) equals to \( \bar{A}_R \), and satisfies \( A' \leq A_M < \bar{A}_R \). \( \Delta \) indicates the change in the corresponding value associated with the change from the no-return policy to the full-return policy.

\[
\begin{array}{c|c|c|c|c}
\text{(Retailer’s risk aversion, Manufacturer’s risk aversion)} & \text{(Low, Low)} & \text{(Low, High)} & \text{(High, Low)} & \text{(High, High)} \\
\hline
\Delta w & + & \text{sign}(\alpha_i/\alpha_R) & + & \text{sign}(A_R - A_M) \\
\Delta s & 0 & + & + & + \\
\Delta MS^M & \text{sign}(A_R - A_M) & - & + & \text{sign}(A_R - A_M) \\
\Delta MS^R & - & + & - & + \text{ for } A' \leq A_R < \bar{A}_R \\
& & & & - \text{ for } \bar{A}_R < A_R < 1 \\
\end{array}
\]

From the result in Table 4, we can derive the following propositions. Here, we assume that \( \lambda \leq A' \) holds in all the propositions. For the case of (Low, High) in Table 4, \( A_R \leq A' \leq A_M \) holds. Therefore, it is easy to verify that if \( A_R > A_M \), then \( \Delta MS^M > 0 \). Also, if \( A_R \leq A_M \), then \( \Delta MS^M \leq 0 \). Hence, Proposition 1 holds.

**Proposition 1** (Manufacturer’s preference as to distribution policies):
The manufacturer prefers the full-return policy to the no-return policy if and only if \( A_R > A_M \) (or equivalently, \( k_R > k_M \)).

To understand Proposition 1, first note that the value of \( MS^{M*} \) for the no-return policy is determined by \( k_R \) while that for the full-return policy is determined by \( k_M \). Also note that the functional forms in Figures 4f and 5f are identical, which can be confirmed in the raw of \( MS^{M*} \) in Table 3. Hence, the relative magnitude of \( A_M \) and \( A_R \) (or \( k_R \) and \( k_M \)) determines which distribution policy the manufacturer prefers. For instance, if the risk sensitivity of retailer is greater than the manufacturer’s, the manufacturer prefers the full-return policy to take advantage of his own smaller risk sensitivity. The following proposition regarding the retailer’s preference on the distribution policies can be also verified from Table 4.

**Proposition 2** (Retailer’s preference as to distribution policies):
(a) If the manufacturer is low risk-averse (including risk-neutral), then the retailer prefers the no-return policy to the full-return policy.
(b) If both the manufacturer and retailer are high risk-averse with an additional condition of \( A' \leq A_R < \bar{A}_R \), then the retailer prefers the full-return policy to the no-return policy.

There is an argument that a risk-neutral manufacturer should absorb the retailer’s inventory risk in general. Thus, if the manufacturer is low risk-averse (including
risk-neutral), then the retailer might prefer the full-return policy to the no-return policy. However, Proposition 2a shows that the argument does not hold. Proposition 2a indicates that the manufacturer’s low risk averseness leads to the retailer’s preference of the no-return policy. The reason is as follows. Comparing the left hand side of Figure 5c to 5f and the two sides of Figure 4c to 4f, we can verify that the low risk-averse manufacturer sets the wholesale price \( w \) high for the full-return policy and relatively low for the no-return policy. This difference primarily explains the retailer’s preference of the no-return policy. Proposition 2b specifies the sufficient condition for the retailer to prefer the full-return policy. It requires not only the high risk averseness of manufacturer but also the high risk averseness of retailer with an additional condition. Proposition 2 indicates that the retailer’s preference as to distribution policy is very much related to whether the manufacturer is high or low risk-averse. Finally, the following proposition holds from Table 4.

**Proposition 3** (Win-win situations for the manufacturer and retailer): 
(a) If \( A' \leq A_M < A_R < \Lambda_R \) holds, then both the manufacturer and retailer prefer the full-return policy to the no-return policy. 
(b) If \( A_R < A_M \leq A' \) holds, then both the manufacturer and retailer prefer the no-return policy to the full-return policy.

Proposition 3a states that, in (High, High) case with an additional condition \( A_M < A_R < \Lambda_R \), both the parties prefer the full-return policy to the no-return policy. That is, the full-return policy is the win-win policy in this situation. To understand this, first review Proposition 1, which states that the relative magnitude of \( A_M \) and \( A_R \) determines which distribution policy is preferred by the manufacturer. In this case, the manufacturer prefers the full-return policy to take advantage of his smaller risk sensitivity. Then, Proposition 2b states that the high risk averseness of the two parties along with \( A_R < \Lambda_R \) leads to the retailer’s preference of the full-return policy. That is, with somewhat different reasons, both the parties prefer the full-return policy.

Proposition 3b states that, in (Low, Low) case with an additional condition \( A_R < A_M \leq A' \), both the parties prefer the no-return policy. This can be proved by combining the results of Propositions 1 and 2a. In this case, the manufacturer prefers the no-return policy because of \( A_R < A_M \). The retailer prefers also the no-return policy because the manufacturer is low risk-averse and sets the wholesale price \( w \) relatively low for the no-return policy, which leads to a higher MS value.

In summary, the manufacturer prefers the distribution policy for which the party with a lower risk sensitivity owns the risk. On the other hand, the retailer’s preference is determined by the combination of which phase of risk-averseness each party is in. If both the parties have high risk averseness, then the retailer prefers the full-return policy with a mild condition. If both the parties have low risk averseness, then the retailer prefers the no-return policy. Hence, there exist situations where the two parties’ preferences as to distribution policies can be aligned.

6. **Different implications between Tsay (2002) and this paper**

We have in this paper derived the insights relevant to practice, which have important difference from those in Tsay (2002) although the two papers use the same basic model. Tsay (2002) concludes that the two parties’ preferences of distribution policies are
always in conflict. In other words, the win-win situations stipulated in Proposition 3 in this paper do not exist. This discrepancy stems from the following two reasons.

First, Tsay (2002) considered only the order size $s$ satisfying $\alpha_l / 2 \leq s \leq \alpha_h / 2$ in Stage 4 of the decision sequence protocol for the no-return policy, which corresponds to Figure 1a. That is, the situation of $s \leq \alpha_l / 2$ corresponding to Figure 1b is not considered. As discussed in the section entitled “Phase transition for the no-return policy,” if the retailer’s risk sensitivity $k_R$ becomes greater than $k'$, the equilibrium outcome for the situation corresponding to $s \leq \alpha_l / 2$ becomes valid.

Second, Tsay (2002) did not impose the constraint $q_i \geq 0$ in (6) for the full-return policy. Without the constraint, the wholesale price greater than $\alpha_l / \beta$ leads to a negative profit for the manufacturer when the low demand scenario is realized (See Figure 2b). As a result, for the low phase of manufacturer’s risk sensitivity, the wholesale price for the full-return policy is set lower than it should be. Therefore, the full-return policy becomes less attractive for the low risk-averse manufacturer than that in our consideration.

Due to the above two reasons, Tsay (2002) identifies only the two equilibrium outcomes out of the four in Table 3. The two equilibrium outcomes coincide with that of the low risk-averse retailer for the no-return policy and that of the high risk-averse manufacturer for the full-return policy. This paper thoroughly addresses the interaction of risk sensitivity of the two agents regarding the distribution policies.

7. Conclusion

In this paper, we revisited the issue of how the risk averseness of both the manufacturer and retailer affects their preference for the full-return policy to the no-return policy. Our finding is that it is important to recognize two phases of high and low risk averseness for each of the two, and that their preference largely depends on the combination of the two phases.

In intuitive terms, each agent’s preference for the channel policy should be understood as follows. The manufacturer prefers the full-return policy to the no-return policy if and only if the retailer’s risk sensitivity is greater than the manufacturer’s risk sensitivity. This is consistent with the common expectation that the manufacturer takes advantage of the lower risk averseness of the two associated with the two policies, comparing the retailer’s risk averseness for the no-return policy with the manufacturer’s risk averseness for the full-return policy.

The retailer’s preference is somewhat more intricate, and how the manufacturer’s risk averseness affects its setting of wholesale price plays a key role. If the wholesale price is set relatively high for the full-return policy, then the retailer cannot sell anything in case of low demand realization. Due to the high wholesale price, the manufacturer’s profit becomes either zero or very high. Therefore, the high risk-averse manufacturer tends to set the wholesale price low to avoid the high profit variance situation associated with the full-return policy. With the low wholesale price under the full-return policy, the retailer can enjoy relatively high demand and thus high profit. That is, the retailer prefers the full-return policy only if the manufacturer is high risk averse.

With the above two insights together, the manufacturer’s risk averseness must be high for the retailer to prefer the full-return policy, and the retailer’s risk averseness must be even higher than the manufacturer’s risk averseness for the manufacturer to prefer the full-return policy. Hence, only if both the manufacturer and retailer are high risk averse, the full-return policy can be a viable and sustainable channel policy.
This intricate role of risk averseness may be contributing to a mixed feeling that practitioners have on the value of return policies. Along with the recent finding in the literature on the factors contributing to the preference for the full-return policy to the no-return policy, the intuitive understanding of how risk averseness leads to the sustainable adoption of the full-return policy is important. This suggests that we must recognize more the importance of risk attitudes when we design and evaluate return policies or channel contracts in general.

In this paper, we focused on the full-return and no-return policies that are extreme cases from the viewpoint of risk in order to show the role of risk averseness on the preferences for channel policy. Even for such extreme cases, we show that the role of risk averseness on the adoption of channel policy is complex and intricate. Although it is challenging, this research may be extended to address the partial-return policy that the retailer returns all unsold inventories for a partial credit, or returns certain portion of original order for a full credit. For the conflicting situations identified in this paper, a Pareto-improvement partial-return policy might exist over the no-return policy. This is a topic for further research.

**Appendix**

*Derivation of the optimal wholesale price for \( \lambda \leq \Lambda' \)*

We show the derivation of the optimal wholesale price for the condition \( \lambda \leq \Lambda' \left( 1 - \left( \frac{\alpha_l}{\alpha_h - \alpha_l} \right)^2 \right) \). The threshold probability \( \Lambda' \) satisfies the following three equations:

1. \( \Lambda' = \frac{\alpha_h}{(\alpha_h - \alpha_l)^2}(\alpha_h - 2\alpha_l) \)
2. \( \alpha_h / \alpha_l = 1 + \frac{1}{\sqrt{1 - \Lambda'}} \)
3. \( \Lambda' \alpha_l + (1 - \Lambda')\alpha_h = \alpha_h \alpha_l / (\alpha_h - \alpha_l) \)

The three equations are used for the derivation for the optimal wholesale price.

(a) *Full-return policy*

1. **Assume that \( \Lambda_M \leq \Lambda' \) holds.**

Since \( \alpha_h \geq \left( 1 + \frac{1}{\sqrt{1 - \Lambda_M}} \right) \alpha_l \) and \( \sqrt{1 - \Lambda_M} > 1 - \Lambda_M \) for \( \Lambda_M \leq \Lambda' \), then

\[
MSM \left( \frac{\alpha_M}{2\beta} \right) - MSM \left( \frac{\alpha_h}{2\beta} \right) = \frac{1}{8\beta} \left( \alpha_M + \sqrt{1 - \Lambda_M} \alpha_h \right) \left( \Lambda_M \alpha_l - \left( \sqrt{1 - \Lambda_M} - (1 - \Lambda_M) \right) \alpha_h \right) \leq \frac{1}{8\beta} \left( \alpha_M + \sqrt{1 - \Lambda_M} \alpha_h \right) \left( \Lambda_M \alpha_l - \left( \sqrt{1 - \Lambda_M} - (1 - \Lambda_M) \right) \left( 1 + \frac{\sqrt{1 - \Lambda_M}}{1 - \Lambda_M} \right) \alpha_l \right) = 0.
\]

Thus, \( MSM \left( \frac{\alpha_M}{2\beta} \right) \leq MSM \left( \frac{\alpha_h}{2\beta} \right) \). Since \( \lambda \leq \Lambda' \) implies that \( \alpha_h > 2\alpha_l \), then \( \frac{\alpha_l}{\beta} < \frac{\alpha_h}{2\beta} < \frac{\alpha_h}{\beta} \).

Therefore, the optimal wholesale price \( w^* = \frac{\alpha_h}{2\beta} \).

2. **Assume that \( \Lambda' \leq \Lambda_M \) holds.**

Since \( \alpha_h \leq \left( 1 + \frac{1}{\sqrt{1 - \Lambda_M}} \right) \alpha_l \) and \( \sqrt{1 - \Lambda_M} > 1 - \Lambda_M \) for \( \Lambda' \leq \Lambda_M \), then

\[
MSM \left( \frac{\alpha_M}{2\beta} \right) - MSM \left( \frac{\alpha_h}{2\beta} \right) = \frac{1}{8\beta} \left( \alpha_M + \sqrt{1 - \Lambda_M} \alpha_h \right) \left( \Lambda_M \alpha_l - \left( \sqrt{1 - \Lambda_M} - (1 - \Lambda_M) \right) \alpha_h \right) \geq \frac{1}{8\beta} \left( \alpha_M + \sqrt{1 - \Lambda_M} \alpha_h \right) \left( \Lambda_M \alpha_l - \left( \sqrt{1 - \Lambda_M} - (1 - \Lambda_M) \right) \left( 1 + \frac{1}{\sqrt{1 - \Lambda_M}} \right) \alpha_l \right) = 0.
\]

Thus, \( MSM \left( \frac{\alpha_M}{2\beta} \right) \geq MSM \left( \frac{\alpha_h}{2\beta} \right) \). Since \( \alpha_M \leq \alpha_h \alpha_l / (\alpha_h - \alpha_l) \) for \( \Lambda' \leq \Lambda_M \) and \( \alpha_h > 2\alpha_l \) for the condition \( \lambda \leq \Lambda' \), then \( \frac{\alpha_l}{\beta} > \frac{\alpha_M}{2\beta} = \frac{1}{2\beta} \left( 2\alpha_l - \alpha_M \right) = \frac{1}{2\beta} \left( 2\alpha_l - \alpha_h \alpha_l / (\alpha_h - \alpha_l) \right) = \frac{\alpha_l}{2\beta (\alpha_h - \alpha_l)} (\alpha_h - 2\alpha_l) > 0 \). Thus, it satisfies \( 0 < \frac{\alpha_M}{2\beta} < \frac{\alpha_l}{\beta} \).
Therefore, the optimal wholesale price \( w^* = \frac{\overline{a}_M}{2\beta} \).

(b) No-return policy

(b-1) Assume that \( \Lambda_R \leq \Lambda' \) holds.

Since \( a_h \geq \left( 1 + \frac{1}{1-\Lambda_R} \right) a_t \) and \( \sqrt{1-\Lambda_R} > 1 - \Lambda_R \) for \( \Lambda_R \leq \Lambda' \), then
\[
\Pi^M \left( \frac{(1-\Lambda_R)a_h}{2\beta} \right) - \Pi^M \left( \overline{a}_R \right) = \frac{1}{\beta} \left( \alpha_R + \sqrt{1-\Lambda_R} a_h \right) \left( -\Lambda_R a_t + \left( \sqrt{1-\Lambda_R} - (1 - \Lambda_R) \right) a_h \right) \geq 0.
\]

Thus, \( \Pi^M \left( \frac{(1-\Lambda_R)a_h}{2\beta} \right) \geq \Pi^M \left( \overline{a}_R \right) \). Since \( \lambda \leq \Lambda' \) implies that \( a_h > 2a_t \), then \( \frac{(1-\Lambda_R)a_h}{2\beta} < \frac{(1-\Lambda_R)(a_h-a_t)}{\beta} \). Therefore, the optimal wholesale price \( w^* = \frac{(1-\Lambda_R)a_h}{2\beta} \).

(b-2) Assume that \( \Lambda' \leq \Lambda_R \) holds.

Since \( a_R \leq \left( 1 + \frac{1}{1-\Lambda_R} \right) a_t \) and \( \sqrt{1-\Lambda_R} > 1 - \Lambda_R \) for \( \Lambda' \leq \Lambda_R \), then
\[
\Pi^M \left( \frac{(1-\Lambda_R)a_h}{2\beta} \right) - \Pi^M \left( \overline{a}_R \right) = \frac{1}{\beta} \left( \alpha_R + \sqrt{1-\Lambda_R} a_h \right) \left( -\Lambda_R a_t + \left( \sqrt{1-\Lambda_R} - (1 - \Lambda_R) \right) a_h \right) \leq 0.
\]

Thus, \( \Pi^M \left( \frac{(1-\Lambda_R)a_h}{2\beta} \right) \leq \Pi^M \left( \overline{a}_R \right) \). Since \( 1 - \frac{(a_t)}{(a_h-a_t)} \leq \Lambda_R \) for \( \Lambda' \leq \Lambda_R \) and \( a_h > 2a_t \) for the condition \( \lambda \leq \Lambda' \), then \( \overline{a}_R \frac{(1-\Lambda_R)(a_h-a_t)}{\beta} = \frac{1}{2\beta} \left( a_t - (1 - \Lambda_R) (a_h - a_t) \right) \geq \frac{1}{2\beta} \left( a_t - \frac{(a_t)}{(a_h-a_t)} (a_h - 2a_t) \right) > 0 \). Thus, it satisfies \( \frac{(1-\Lambda_R)(a_h-a_t)}{\beta} < \frac{\overline{a}_R}{\beta} < \frac{\overline{a}_R}{\beta} \).

Therefore, the optimal wholesale price \( w^* = \frac{\overline{a}_R}{2\beta} \).

\[\textbf{Derivation of the optimal wholesale price for } \lambda > \Lambda'\]

We show the derivation of the optimal wholesale price for the condition \( \lambda > \Lambda' \left( = 1 - \left( \frac{a_t}{a_h-a_t} \right)^2 \right) \).

(c) Full-return policy

Since \( a_h \leq \left( 1 + \frac{1}{1-\Lambda_M} \right) a_t \) and \( \sqrt{1-\Lambda_M} > 1 - \Lambda_M \) for \( \Lambda' < \lambda \leq \Lambda_M \), \( MS^M \left( \frac{\overline{a}_M}{2\beta} \right) \geq MS^M \left( \frac{a_h}{2\beta} \right) \). (See the proof of (a-2))

(c-1) Assume that \( \Lambda' \leq 0 < \lambda \) holds.

Since \( \Lambda' = 1 - \left( \frac{a_t}{a_h-a_t} \right)^2 \leq 0 \), then \( a_h \leq 2a_t \). Thus, \( \frac{a_t}{\beta} - \frac{\overline{a}_M}{2\beta} = \frac{1}{2\beta} (2a_t - \overline{a}_M) \geq 0 \). Thus, it satisfies \( 0 < \frac{\overline{a}_M}{2\beta} < \frac{a_t}{\beta} \).

(c-2) Assume that \( 0 < \Lambda' < \lambda \) holds.

Since \( \Lambda' = 1 - \left( \frac{a_t}{a_h-a_t} \right)^2 \geq 0 \), then \( a_h > 2a_t \). Thus, \( \frac{a_t}{\beta} - \frac{\overline{a}_M}{2\beta} = \frac{1}{2\beta} (2a_t - \overline{a}_M) \geq 0 \). Thus, it satisfies \( 0 < \frac{\overline{a}_M}{2\beta} < \frac{a_t}{\beta} \).

Therefore, from (c-1) and (c-2) the optimal wholesale price \( w^* = \frac{\overline{a}_M}{2\beta} \).
(d) No-return policy
Since $\Lambda' < \lambda \leq \Lambda_R$, $\Pi^M \left(\frac{(1-\Lambda_R)\alpha_h}{2\beta}\right) \leq \Pi^M \left(\frac{\pi_R}{2\beta}\right)$ (See the proof of (b-2))

(d-1) Assume that $\Lambda' \leq 0 < \lambda$ holds.

Since $\Lambda' = 1 - \left(\frac{\alpha_i}{\alpha_h-\alpha_i}\right)^2 \leq 0$, then $\alpha_h \leq 2\alpha_i$. Thus, $\frac{\pi_R}{2\beta} - \frac{(1-\Lambda_R)(\alpha_h-\alpha_i)}{\beta} \geq \frac{1}{2\beta} (\alpha_i \Lambda_R) > 0$. Thus, it satisfies $\left(\frac{(1-\Lambda_R)(\alpha_h-\alpha_i)}{\beta}\right) < \frac{\pi_R}{2\beta} < \frac{\pi_R}{\beta}$.

(d-2) Assume that $0 < \Lambda' < \lambda$ holds.

Since $\Lambda' = 1 - \left(\frac{\alpha_i}{\alpha_h-\alpha_i}\right)^2 > 0$, then $\alpha_h > 2\alpha_i$. Thus, $\frac{\pi_R}{2\beta} - \frac{(1-\Lambda_R)(\alpha_h-\alpha_i)}{\beta} \geq \frac{1}{2\beta} (\alpha_i - (1 - \Lambda_R)(\alpha_h - \alpha_i)) \geq \frac{\alpha_i}{2\beta} (\alpha_h - 2\alpha_i) > 0$. Thus, it satisfies $\left(\frac{(1-\Lambda_R)(\alpha_h-\alpha_i)}{\beta}\right) < \frac{\pi_R}{2\beta} < \frac{\pi_R}{\beta}$.

Therefore, from (d-1) and (d-2) the optimal wholesale price $w^* = \frac{\pi_R}{2\beta}$.

Derivation of Table 4

Low risk-averse retailer and High risk-averse manufacturer $\Lambda_R \leq \Lambda'$ and $\Lambda' \leq \Lambda_M$

Proof of $\Delta(MS^M) \leq 0$: Since $\Lambda_R \leq \Lambda' \leq \Lambda_M$, then $-(\sqrt{1-\Lambda_R} - (1-\Lambda_M)) < 0$ and $\alpha_h \geq \left(1 + \frac{1}{\sqrt{1-\Lambda_R}}\right)\alpha_i$. For these condition, it is easy to verify $\Delta(MS^M) \leq 0$.

Proof of $\Delta(MS^R) \geq 0$: It is easy to verify that $\Delta(MS^R)$ is strictly decreasing with respect to $\Lambda_R$ for $\Lambda_R \leq \Lambda' \leq \Lambda_M$. If $\Lambda_R = \Lambda'$ holds, then $\Delta(MS^R) \geq 0$ for $\Lambda' \leq \Lambda_M$. Therefore, $\Delta(MS^R) \geq 0$ for $\Lambda_R \leq \Lambda' \leq \Lambda_M$.

High risk-averse retailer and High risk-averse manufacturer $\Lambda' \leq \Lambda_R$ and $\Lambda' \leq \Lambda_M$

Proof of $\Delta(MS^R)$: It is easy to verify that $\Delta(MS^R)$ is strictly decreasing with respect to $\Lambda_R$ for $\Lambda' \leq \Lambda_R$ and $\Lambda' \leq \Lambda_M$. Since $\Lambda' \leq \Lambda_M$, then it is easy to verify $\lim_{\Lambda_R \to 1} \Delta(MS^R) = 0$. If $\Lambda_R = \Lambda_M$ holds, then $\Delta(MS^R) > 0$. Thus, there exists uniquely $\bar{\Lambda}_R$ such that $\Delta(MS^R) = 0$ when $\Lambda_R$ equals to $\bar{\Lambda}_R$ and it satisfies $\Lambda_M < \bar{\Lambda}_R$.

References
Chen, J., Bell, P.C., 2011. Coordinating a decentralized supply chain with customer returns and


