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Disclosure policy in a mixed market*

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ABSTRACT
This paper investigates the optimal disclosure strategy for private information in a mixed duopoly market, where a state-owned enterprise (SOE) and a joint-stock company compete to supply products. I construct a model where the two firms compete in either quantity or price, and uncertainty is associated with either marginal cost or market demand. The model identifies the optimal disclosure strategies that constitute a perfect Bayesian equilibrium by type of competition and uncertainty. In Cournot competition, both firms disclose information under cost uncertainty, while only the SOE or neither firm discloses information under demand uncertainty. Alternatively, in Bertrand competition, only the joint-stock company discloses information under cost uncertainty or demand uncertainty. Recently, developed countries have required the same level of disclosure standards for SOEs as for ordinary joint-stock companies. The findings described in this paper warn that such mandatory disclosure by SOEs can trigger a reaction by joint-stock companies, putting the economy at risk of a reduction in welfare.

Keywords: Disclosure; Cost information; Market information; Mixed duopoly

JEL classifications: M41, M48, H83

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1. Introduction

In both developed and developing countries, numerous state-owned enterprises (SOEs) set up by governments coexist and compete with joint-stock companies in supplying products across a wide range of industries. Industries currently characterized by such 'mixed markets' include railways, airlines, telecommunications, natural gas, electricity, automobiles, and steel, along with services such as banking, mortgage loans, healthcare, life insurance, hospitals, television broadcasting, and education. Accordingly, economic studies concerning mixed markets have become increasingly popular (e.g., De Fraja and Delbono, 1990; Matsumura, 1998; Matsumura and Matsushima, 2004; Ishibashi and Matsumura, 2006). However, this issue has not commanded sufficient attention in the accounting literature. There has been particularly little consideration of the information aspects of mixed markets. Because previous analytical accounting studies on information disclosure have proved that an information-sharing strategy by competitors crucially affects equilibrium consequences, disclosure policy by an SOE in a mixed market can also affect market outcomes, such as the level of social welfare. Nonetheless, no previous study has examined the economic effects of information disclosure by an SOE.¹

Recently, developed countries have required the same level of disclosure standards for SOEs as for ordinary joint-stock companies. For example, SOEs in Sweden must submit quarterly reports, while most OECD countries only require biannual reports from SOEs. In the UK, SOEs established as companies under the Companies Act have the same reporting requirements as all other companies registered under the same legislation. Given the tendency for disclosure standards on SOEs to become stricter, the objective of this study is to analyze whether such increased disclosure requirements are desirable when SOEs compete with joint-stock companies. To achieve this objective, I construct a model where an SOE and a joint-stock company compete in either quantity or price, and uncertainty is associated with either marginal cost or market demand.² After calculating the equilibrium in this mixed duopoly by type of competition and uncertainty, I identify optimal disclosure strategies that constitute a perfect Bayesian equilibrium (PBE). In Cournot competition, both firms disclose information under cost uncertainty, while only the SOE or neither firm discloses information under demand uncertainty. Alternatively, in Bertrand competition, only the joint-stock company discloses information under either cost uncertainty or demand uncertainty. These results warn that recent mandatory disclosure by SOEs can trigger a reaction by joint-stock companies, putting the economy at risk of a decrease in welfare.

The above results relate to those found in a normal duopoly between profit-maximizing firms, as revealed in earlier disclosure models. To date, there has been an extensive literature investigating the economic outcomes of private information sharing

¹ Note that the SOE and the joint-stock company are often referred to, respectively, as the 'public firm' and the 'private firm' in the economic literature.
² While I consider two types of uncertainty (i.e., cost uncertainty and demand uncertainty), both of which are especially important to competing firms, note that the existing literature suggests that a more general distinction between the two types of uncertainty relates to the private and common values for the firms. See Raith (1996) for a detailed explanation.
between oligopolistic firms (e.g., Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Li, 1985). Subsequent to this economic work, numerous contributions from an accounting viewpoint have also appeared. Most of these models analyzing duopoly or oligopoly settings are to establish which type of competition (either Cournot or Bertrand) makes a significant difference to the ex ante choice of sharing information on demand or costs (e.g., Darrough, 1993; Pae, 2002; Hughes and Williams, 2008; Rothenberg, 2009; Arya et al., 2010; Bagnoli and Watts, 2010, 2013; Suijs and Wielhouwer, 2014). Note that there are two representative studies in this line of accounting that the structure of my model significantly depends upon: the first is Darrough (1993), and the second is Suijs and Wielhouwer (2014).

Apart from the disclosure literature, substantial previous work on the mixed market issue also appears. Merrill and Schneider (1966), a pioneering study in mixed oligopolies, and Bös (1986, 1991), De Fraja and Delbono (1990), and Nett (1993) provide comprehensive surveys of previous work in this field. Nearly all of these preceding theoretical studies employ a classical oligopoly model framework to investigate mixed oligopoly. More specifically, these studies rely on the assumption that the SOE maximizes social welfare, comprising both consumer surplus and the combined profits of both the SOE and the joint-stock company, whereas the joint-stock company maximizes only its own profit.

The survey of both the accounting literature and the economic literature reveals that no analysis has yet examined optimal disclosure policy for SOEs and its potential effect on social welfare. This paper demonstrates that increasing disclosure requirements for SOEs may not always be a wise decision because such requirements may undermine social welfare. This policy implication is noteworthy and constitutes the major contribution of this study to the accounting literature.

The remainder of the paper is structured as follows. In Section 2, I provide an overview of SOEs and the disclosure regulations that apply to these organizations. Section 3 describes the assumptions and settings of my disclosure model. Given these settings, I derive the relevant equilibrium under cost uncertainty in Section 4 and under demand uncertainty in Section 5. I also explore the logic that leads to the equilibrium results in these two sections. Section 6 provides some concluding remarks.

2. State-owned Enterprises and Disclosure Regulations

Because the purpose of this paper is to analyze the disclosure policies chosen by an SOE in addition to a joint-stock company, in this section I provide an overview of SOEs and disclosure regulations for them. I also clarify the similarities and differences between the regulations for SOEs and those for joint-stock companies. Because SOEs are financed differently from joint-stock companies, they may use other ways to communicate with their stakeholders such as private communication channels to ensure accountability. Nevertheless,

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3 See Verrecchia (2001), Dye (2001), and Beyer et al. (2010) for comprehensive reviews of the disclosure policy literature.
the most important fact identified in this section is that governments in developed economies have recently required almost the same method and level of disclosure regulations for SOEs as for ordinary joint-stock companies.

2.1. State-owned Enterprises

The SOE assumed in my model should satisfy the following three conditions: (i) it is owned by the government, (ii) it competes with a profit-maximizing firm, and (iii) it does not pursue its own profit. I first confirm that SOEs that meet these three conditions actually exist in real-world economies before constructing the mixed market model.

In the broadcasting sector, the British Broadcasting Corporation (BBC) in the UK, which is funded by the UK government and has an educational mission, is a typical example of a firm that competes with joint-stock companies and does not have profit maximization as its main objective. Channel 4 in the UK is also a publicly owned and not-for-profit broadcaster that competes with private channels such as ITV, which is the largest commercial broadcasting company in the UK. Other than these broadcasters in the UK, the Swiss Broadcasting Corporation and ZDF in Germany are also government-owned not-for-profit broadcasting companies that compete with private broadcasters. In health insurance, the VHI (Voluntary Health Insurance) in Ireland is a statutory corporation and a not-for-profit organization. Moreover, airport operations in Norway are characterized by a mixed market:

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4 George and La Manna (1996) state that the BBC is committed to minimum production targets in terms of drama, documentaries, and news coverage. Indeed, the Department for Culture, Media, and Sport (2006: 2) documents 'the BBC’s public nature and its objects' as '(1) The BBC exists to serve the public interest. (2) The BBC's main object is the promotion of its Public Purposes.' If the objective of the BBC were not public welfare but profit maximization, the company would broadcast a wider variety of programs to compete with private channels.

5 Channel 4 (2014) states: 'Channel 4 is a publicly-owned, commercially-funded public service broadcaster. ... Non-executive directors are appointed by OFCOM in agreement with the Secretary of State for Culture, Media and Sport. This system ensures our not-for-profit status...'

6 SRGSSR (2012) states: 'SRG is a media enterprise governed by private law and operated in accordance with the principles of company law. Its remit is based on the Swiss Federal Constitution, the Federal Radio and Television Act (RTVA) and its charter, and is one of public service. As a non-profit organisation, SRG derives around 70 percent of its revenues from license fees and about 30 percent from commercial activities.' Moreover, Legislative Council of the Hong Kong (2006) refers to the ZDF as being: 'Established in 1963 as an independent non-profit-making corporation under the authority of all the federal states of Germany.'

7 VHI (2007) states: '... There is no assessment of the positive impact on consumers of the not for profit status of Vhi Healthcare...' In addition, Civitas (2013) notes: 'In 1957, the government established the Voluntary Health Insurance (VHI) Board, a non-profit state-owned insurance company whose board members were appointed by the Department of Health ... In the 1990s, partly due to concerns that the VHI's dominance was breaching EU competition law, the insurance market was opened to greater competition, after which a variety of private providers increased their market share, though in 2006 the VHI represented
Avinor is a state-owned not-for-profit company that operates a significant proportion of civil airports in Norway.\(^8\) Finally, mixed markets are seen in the railway industry: V/Line is a government-owned and not-for-profit regional passenger train and coach operator in the State of Victoria in Australia, where private rail transport operators (e.g., Great Southern Rail) also exist.\(^9\) This overview suggests that not-for-profit SOEs operating in mixed markets prevail in a wide range of industries and countries both inside and outside Europe.

2.2. Disclosure Regulations

Surveys by OECD (2005, 2011) provide comprehensive perspectives on the disclosure rules applied to SOEs in developed economies including European countries. Even though OECD (2005) indicates that details of the disclosure rules vary among the European countries, it should be noted that similar levels of disclosure standards are applied overall to SOEs as compared with joint-stock companies. The following statement in OECD (2005: 98) confirms this fact: 'Financial disclosure by SOEs has improved significantly in recent years in many OECD countries. Generally speaking, SOEs have to report the same way as public companies since they are also subject to Company Law. Among the countries surveyed, there are no cases where SOEs have been subject to less stringent standards than applied to ordinary public company regarding disclosure and transparency. On the contrary, in most cases, SOEs are subject to additional requirements.'

As an example that represents this statement, OECD (2005) suggests that Sweden applies more stringent disclosure standards to SOEs than to listed joint-stock companies. In Sweden, the ownership entity has a degree of discretion about some specific aspects of financial reporting by SOEs based on the notion that their ultimate owner is the general public, so that they are even more 'public' than joint-stock companies. This principle is embodied in Regeringskansliet (2002: 11) as: 'For state-owned companies, the requirement for transparency is a question of democracy since the companies are ultimately owned by the Swedish people. The government therefore considers that these companies should be at least as transparent as listed companies.' Furthermore, in Sweden SOEs have to submit quarterly reports, whereas in most OECD countries SOEs publish biannual reports. In the UK also, SOEs set up as companies under the Companies Act have the same reporting requirements as all other registered companies as set out in the Companies Act, and in conformity with UK accounting standards.\(^10\) In addition, OECD (2011) reports the change and reform of the disclosure rules of SOEs in OECD countries from 2005 to 2011, showing that all changes

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\(^7\)5% of the market and in December 2009 it still had a 65.5% share.'
\(^8\) Moody's (2013) discusses: 'Avinor AS benefits from a near monopoly position in Norway, accounting for over 90% of Norwegian passenger traffic in 2012. It is currently 100% owned by the Kingdom of Norway.' Moreover, Danske Bank (2013) states: 'Avinor AS is owned by the Norwegian government and is characterised as a sector political investment with socio-political targets rather than a profit-maximising business model.'
\(^9\) V/Line (2013) states: 'As a not-for-profit operator, V/Line is fully owned by the State of Victoria, with train services running between Melbourne and the following townships...'.
\(^10\) See OECD (2005: 98) for details of Swedish and UK cases.
involve greater disclosure. For example, Germany has strengthened the accountability mechanisms around annual reporting. These disclosure systems in the respective countries further validate the OECD statement.

Finally, it should be noted that OECD (2005: 214) documents the following guideline: 'SOEs should be subject to the same high quality accounting and auditing standards as listed companies. Large or listed SOEs should disclose financial and non-financial information according to high quality internationally recognised standards.' Such a policy recommendation is expected to promote the application of more stringent disclosure standards to SOEs. At first glance, it seems that such strict disclosure regulations for an SOE are favorable, because domestic citizens are ultimately the owners of the enterprise and are thus entitled to receive intrafirm information. However, results in this study are contrary to intuition and the recent regulatory trend. If an SOE is not a monopolistic firm and a joint-stock company operates in the same sector, the imposition of stringent disclosure standards on the SOE can reduce social welfare of the country by triggering reactions by the joint-stock company that competes with the SOE. I start to prove this possibility from the following section.

3. Assumptions and Settings

I consider a mixed duopoly market where an SOE (Firm 0) and a joint-stock company (Firm 1) compete to supply a differentiated product. Firm i (i = 0, 1) produces brand i of the product at a variable cost of \( c + k_i \) per unit, with no fixed cost. Henceforth, \( i \) represents either 0 or 1 when the variable is present alone. While both firms know the value of \( c \), only Firm \( i \) knows the value of \( k_i \) as private information. I assume \( k_i \) is a normally distributed random variable with a mean of 0 and variance of \( s_i \).

The utility function of a representative consumer buying the product is:

\[
U(q_0, q_1) = (a + e)(q_0 + q_1) - (q_0^2 + q_1^2 + 2a q_0 q_1)/2, \tag{1}
\]

where \( a (> c) \) is a deterministic positive constant, and \( q_i \) is the quantity consumed of brand \( i \). The variable \( e \) is also a constant, but only the consumer knows its value. For the firms, \( e \) is a random variable that is assumed to be normally distributed with mean 0 and variance \( V \).

The variable \( \theta \in (0, 1) \) represents the degree of substitution between the two products, such that the products become more differentiated as \( \theta \) decreases and approaches zero, and more substitutable as \( \theta \) increases and approaches unity.

\[\text{11} \]

Although \( c+k_i \) or \( a+e \) may take negative values given the assumption of normality, this specification has been conventionally used in the past literature (e.g., Vives, 1984) because it helps simplify the analysis and allows for closed-form solutions. Indeed, one can make the probability that \( c+k_i \) or \( a+e \) is negative arbitrarily small by appropriately choosing the variances of the model. Following the literature, I assume a large \( c \) relative to \( s_i \) and a large \( a \) relative to \( V \) and \( \sigma_e \), such that the probability of negative cost or demand is negligible.
Based on these arrangements, I introduce both a quantity function and a price function to investigate both Cournot competition and Bertrand competition, because previous disclosure studies analyze both competition modes (e.g., Darrough, 1993; Suijs and Wielhouwer, 2014). Let \( p_i \) denote the retail price of brand \( i \). Because the consumer maximizes its own surplus, \( U(q_0, q_1) = (p_0q_0 + p_1q_1) \), with respect to \( q_0 \) and \( q_1 \), I obtain the following linear stochastic inverse demand function for brand \( i \):\(^{12}\)

\[
p_i = a + e - (q_i + \theta q_j).
\]

Henceforth, \((i, j)\) represents either \((0, 1)\) or \((1, 0)\) when the two variables are simultaneously present, but not each alone. Jointly solving the inverse demand equations yields a solution for each quantity as a function of prices:

\[
q_i = \left((1 - \theta) \left(a + e - p_i + \theta p_j\right)\right)/\left((1 - \theta)(1 + \theta)\right).
\]

Although \( e \) is unknown to the firms, each firm forecasts its value by using firm-specific information-gathering skills. Specifically, the firms respectively receive noisy signals of \( e \); the signal observed by Firm \( i \) is \( f_i \) and equals the following:

\[
f_i = e + \varepsilon_i,
\]

where \( \varepsilon_i \) is the noise of the observed signal and follows a normal distribution, independent of \( e \) and \( \xi_i \), with mean 0 and variance \( \sigma_i \). I define that signal \( f_i \) gives more precise information about \( e \) than signal \( f_j \) if the variance of the former, \( \sigma_i \), is smaller than that of the latter, \( \sigma_j \). Therefore, as \( \sigma_i \) ranges from 0 to \( \infty \), the signal goes from being perfectly informative to being not informative at all. That is, when the information is perfect, \( E[e|f_i] = f_i \), but when there is no information, \( E[e|f_i] = 0 \). I assume all parameters in the model, except information, are common knowledge for both firms.

Firm 0 operates to maximize social welfare, while Firm 1 aims to maximize its own profit. Consumer surplus is:

\[
CS = U(q_0, q_1) - (p_0q_0 + p_1q_1).
\]

\(^{12}\) Following previous studies that consider the information-sharing problem overviewed in the introduction, I confine the demand schedule to a linear form primarily because I construct a stochastic duopoly model. If I introduced a more general, say, nonlinear demand function, the expected value of the objective functions could not be algebraically described using the parameters for variances (i.e., \( \sigma_i \) and \( \sigma_j \)) because the degree of the random variable terms included in the objective functions could exceed two.
The profit for each firm is:

$$\pi_i = (p_i - c - k_i)q_i.$$  \hfill (6)

From Equations (5) and (6), I calculate social welfare as:

$$SS = CS + (\pi_0 + \pi_1) = U(q_0, q_1) - (p_0q_0 + p_1q_1) + (\pi_0 + \pi_1) = U(q_0, q_1) - ((c + k_0)q_0 + (c + k_1)q_1).$$  \hfill (7)

Because the SOE aims to maximize social welfare, the objective function for Firm 0 is given in Equation (7) and can be rewritten as:

$$SS = (a + e - c - k_0)q_0 + (a + e - c - k_1)q_1 - \left( q_0^2 + q_1^2 + 2\theta q_0q_1 \right)/2,$$  \hfill (8)

while that for Firm 1 is:

$$\pi_1 = (a + e - (q_1 + \theta q_0) - c - k_1)q_1.$$  \hfill (9)

By substituting Equation (3) into Equations (8) and (9), I can restate them as functions of prices rather than quantities:

$$SS = \frac{(p_0 - p_1)(2k_0 - k_1) - p_0 + p_1 + 2(1 - \theta)(a + e)(a + e - 2c - k_0 - k_1) + c(p_0 + p_1) + k_0p_0 + k_1p_1 - p_0p_1)}{2(1 - \theta)(1 + \theta)},$$  \hfill (10)

$$\pi_1 = \frac{(p_1 - c - k_1)(1 - \theta)(a + e) - p_1 + \theta p_0)}{(1 - \theta)(1 + \theta)}. \hfill (11)

The event timeline is as follows. At Date 1, each firm decides its disclosure policy from the binary choices of either fully disclosing information (strategy D) or not disclosing information at all (strategy ND) to the other firm. In the cost uncertainty case, let $\kappa_i$ denote publicly available information for both the firms, whereas $k_i$ is private information that only Firm $i$ knows. To describe the general disclosure policy, I define $\kappa_i = k_i + \delta_i$, where $\delta_i$ is a normally distributed noise variable with variance $\Delta_i$. These notations indicate that $\Delta_i = 0$ and $\kappa_i = k_i$ hold if Firm $i$ discloses its cost information (strategy D). By contrast, if Firm $i$ does not disclose its cost information, then $\Delta_i \to \infty$ (strategy ND). Meanwhile, in the demand uncertainty case, let $\rho_i$ denote publicly available information for both firms, whereas $f_i$ is private information that only Firm $i$ knows. Accordingly, $\rho_i = f_i + \delta_i$ holds. These notations indicate that if Firm $i$ discloses its demand information (strategy D), then $\Delta_i = 0$ and $\rho_i = f_i$ hold. By contrast, if Firm $i$ does not disclose its demand information (strategy ND), then $\Delta_i \to \infty$. After committing to a particular disclosure policy at Date 1, each firm observes a signal.
of information at Date 2. At Date 3, each firm either discloses or does not disclose its signal based on the disclosure policy predetermined at Date 1. Finally, the firms choose their strategic variables at Date 4 and realize profits at Date 5.

Before proceeding to the analysis, it should be noted that Suijs and Wielhouwer (2014) identify two informational effects of disclosure that arise in a duopoly: (i) a strategic information effect and (ii) a market information effect. The strategic information effect refers to information on the firm's quantity/price strategy that disclosure reveals to its competitor. Meanwhile, the market information effect refers to information on the competitor's demand/cost uncertainty that disclosure reveals to this competitor. In the cost uncertainty setting, only the strategic information effect plays a role, while in the demand uncertainty setting, both the strategic information effect and the market information effect play a role. These effects play a crucial role within the mechanism of the model, as I will later elaborate upon when providing a rationale for the analytical results.

4. Model with Cost Uncertainty

4.1. Equilibrium Results

Following previous studies on disclosure policy under oligopoly, I first focus on cost uncertainty; that is, I assume \( e \) is deterministically 0 such that there is no uncertainty associated with demand throughout this section. Because \( k_i \) is random under cost uncertainty, each firm maximizes the expected value of the objective function based on the signal received at Date 4. Given this uncertainty, a strategy for a firm is a function that specifies an action for each possible signal that the firm may receive. I derive the Bayesian–Nash equilibrium (BNE) that specifies pairs of strategies such that each firm's strategy is a best response to the strategy of its rival. Note that the following equilibrium analysis is confined to strategies that are linear in the information variables.

The following lemmas show that there is a unique BNE with linear (affine) strategies.\(^{13}\) The appendix includes all proofs. For notational simplicity, hereafter I use a hat (\(^\hat{}\)) to identify the BNE.

**Lemma 1** Under Cournot competition, the equilibrium quantity choices satisfy:

\[
\hat{q}_0 = \frac{1}{2-\theta^2} \left( (2-\theta)(a-c) - \frac{\theta^2 s_0}{s_0 + \Delta_0} \kappa_0 + \frac{\theta k_i}{s_1 + \Lambda_1} \kappa_1 \right) - k_0
\]

\[
\hat{q}_1 = \frac{1}{2-\theta^2} \left( (1-\theta)(a-c) + \frac{\theta k_0}{s_0 + \Delta_0} \kappa_0 - \frac{\theta^2 s_1}{2(s_1 + \Lambda_1)} \kappa_1 \right) - \frac{1}{2} k_1,
\]

\(^{13}\) The author is very grateful to the editor and an anonymous reviewer for suggesting the precise derivation process for the equilibrium strategies in the appendix and the concise functional forms in Lemmas 1, 2, 3, and 4 that summarize all combinations of the disclosure strategies taken by the two firms.
resulting in expected payoffs to Firm 0 and Firm 1 of:

\[
E(SS(\hat{q}_0, \hat{q}_1)) = \frac{1}{2(1-\theta^2)} \left[ (7 - 6\theta - 2\theta^2 + 2\theta^3)(a-c)^2 + \frac{(4-\theta^2)s_0^2}{s_0 + \Delta_0} + \frac{(3-\theta^2)s_1^2}{s_1 + \Delta_1} \right] \\
+ \frac{s_0\Delta_0}{2(s_0 + \Delta_0)} + \frac{3s_1\Delta_1}{8(s_1 + \Delta_1)} \\
E(\pi_1(\hat{q}_0, \hat{q}_1)) = \frac{1}{(2-\theta^2)^2} \left[ (1-\theta)^2(a-c)^2 + \frac{\theta^2 s_0^2}{s_0 + \Delta_0} + \frac{s_0^2}{s_1 + \Delta_1} \right] + \frac{s_1\Delta_1}{4(s_1 + \Delta_1)}.
\]

**Lemma 2** Under Bertrand competition, the equilibrium price choices satisfy:

\[
\hat{p}_0 = \frac{1}{2-\theta^2} \left[ \theta(1-\theta)a + (2-\theta)c + \frac{\theta^2 s_0}{s_0 + \Delta_0} \kappa_0 - \frac{\theta_0 k_1}{s_1 + \Delta_1} \right] + k_0 \\
\hat{p}_1 = \frac{1}{2-\theta^2} \left[ (1-\theta)a + (1+\theta-\theta^2)c + \frac{\theta_0 k_0}{s_0 + \Delta_0} \kappa_0 - \frac{\theta^2 s_1}{2(s_1 + \Delta_1)} \kappa_1 \right] + \frac{1}{2} k_1,
\]

resulting in expected payoffs to Firm 0 and Firm 1 of:

\[
E(SS(\hat{p}_0, \hat{p}_1)) = \frac{(7 + 7\theta - 7\theta^2 + 2\theta^3)(a-c)^2}{2(1+\theta)(2-\theta^2)^2} + \frac{1}{2(1-\theta^2)} \left[ \frac{s_0\Delta_0}{s_0 + \Delta_0} + \frac{3s_1\Delta_1}{4(s_1 + \Delta_1)} \right] \\
+ \frac{1}{2(1-\theta^2)(2-\theta^2)^2} \left[ \frac{(4-5\theta^2 + 2\theta^4)s_0^2}{s_0 + \Delta_0} + \frac{(3-3\theta^2 + \theta^4)s_1^2}{s_1 + \Delta_1} \right] \\
E(\pi_1(\hat{p}_0, \hat{p}_1)) = \frac{1}{(1-\theta^2)(2-\theta^2)^2} \left[ (1-\theta)^2(a-c)^2 + \frac{\theta^2 s_0^2}{s_0 + \Delta_0} + \frac{s_0^2}{s_1 + \Delta_1} \right] + \frac{s_1\Delta_1}{4(1-\theta^2)(s_1 + \Delta_1)}.
\]

The following propositions immediately follow from Lemmas 1 and 2.

**Proposition 1** Under Cournot competition with cost uncertainty, both the SOE and the joint-stock company disclose their cost information in equilibrium.

**Proposition 2** Under Bertrand competition with cost uncertainty, the joint-stock company discloses its information while the SOE does not disclose its information in equilibrium.

Recall that the previous literature shows that if two competing firms are profit-maximizing firms facing cost uncertainty, they both disclose cost information under quantity competition, whereas they do not disclose information under price competition (e.g., Darrough, 1993). In this respect, Proposition 1 concerning mutual disclosure of cost information under Cournot competition in the mixed-market setting is exactly the same result as the conventional one. In contrast, Proposition 2 suggests that the equilibrium disclosure policy in a mixed market under Bertrand competition is asymmetric in that the joint-stock
company discloses its information while the SOE does not. This is contrary to the conventional finding.

4.2. Rationale

4.2.1. Cournot-type Quantity Competition Case

Because this section considers cost uncertainty, I replace \( e \) in Equations (8) and (9) with 0, yielding:

\[
SS = (a - c - k_0)q_0 + (a - c - k_1)q_1 - \left(q_0^2 + q_1^2 + 2\partial q_0q_1\right)/2, \quad (12)
\]

\[
\pi_1 = (a - (q_1 + \partial q_0) - c - k_1)q_1. \quad (13)
\]

In the absence of uncertainty, Firm 0 maximizes \( SS \) with respect to \( q_0 \) and Firm 1 maximizes \( \pi_1 \) with respect to \( q_1 \). That is, solving \( \partial SS / \partial q_0 = \partial \pi_1 / \partial q_1 = 0 \) for \( q_0 \) and \( q_1 \) yields:

\[
q_0 = \left((2 - \theta)(a - c) - 2k_0 + \theta k_1) / (2 - \theta^2)\right), \quad (14)
\]

\[
q_1 = \left((1 - \theta)(a - c) + \theta k_0 - k_1) / (2 - \theta^2)\right). \quad (15)
\]

I first examine \( SS \), which is the objective function of Firm 0. Rearranging Equation (12) by using the equilibrium quantities of Equations (14) and (15), I obtain the following expression:

\[
SS = \left(1 - \theta^2\right) q_0 - \left((1 - \theta)(a - c) - k_0 + \theta k_1) / (1 - \theta^2)\right)^2 / (2\theta^2) + (a - c)(a - c - k_0 - k_1) / (1 + \theta)
\]

\[
+ \left(k_0^2 + k_1^2\right) / (2(1 - \theta^2)) - \theta k_0k_1 / (1 - \theta^2), \quad (16)
\]

which is concave with respect to \( q_0 \). Therefore, the SOE can increase the expected value of \( SS \) by reducing the variability of this value given the presence of uncertainty.

Recall that the payoff functions for profit-maximizing firms in a normal duopoly are convex with respect to the strategic variables according to previous research (e.g., Darrough, 1993; Suijs and Wielhouwer, 2014). To examine the effects of disclosure, Suijs and Wielhouwer (2014) suggest that when the objective function for a firm is convex and increasing with respect to a decision variable, the effect of the disclosure of good private information dominates the effect of the disclosure of bad private information. Accordingly, they discuss the profit-maximizing firm's optimal disclosure decision when the firm possesses good private information. By contrast, Equation (16) shows that the objective function for the SOE (Firm 0) is concave and increasing in \( q_0 \) when \( q_0 \) is sufficiently small. Therefore, it suffices to discuss the optimal disclosure decision of Firm 0 when it possesses 'bad' private information, for when the objective is concave the effect of disclosure of bad information is
more pronounced than the effect of disclosure of good information.

Furthermore, notice that consumer surplus is a more important component of social welfare that should be increased than producer surplus (i.e., firms' profits) if social welfare maximization is pursued, because social welfare under general market environments is maximized when a perfectly competitive situation is achieved. Consistent with this notion, substituting the reaction function of Firm 0 (Equation (A3) in the appendix) and \( e = 0 \) into the expected price (Equation (2)) conditional on Firm 0's available information yields the following equation:

\[
E(p_0 | k_0, \kappa_0, \kappa_1) = c + k_0, \quad (17)
\]

which indicates that Firm 0 sets the quantity so that the expected price is equal to the marginal cost and its expected profit is thus zero. Note here that the SOE faces the following trade-off. First, for maximizing consumer surplus, the SOE wishes to have prices equal marginal costs or alternatively to maximize total demand \( q_0 + q_1 \) subject to firm profits being nonnegative. Second, the SOE wishes to maintain high prices for maximizing Firm 0 and Firm 1 profits. Based on Equation (17), it seems that the former effect dominates. Hence, I hereafter note that when discussion touches on disclosure effects on social welfare, the SOE wishes to maximize total demand.

Now suppose that Firm 0 receives and discloses 'bad' information; that is, its marginal cost is high. Then, Firm 1 (the joint-stock company) will aggressively increase its quantity to raise its profit, because quantity works as a strategic substitute in Cournot competition.\(^{14}\) The increase in supply by Firm 1 leads to higher consumer surplus. Hence, the best choice for Firm 0 is to disclose bad information in order to induce Firm 1 to increase its supply, which is in line with the SOE's objective to maximize total demand, \( q_0 + q_1 \). That is, Firm 0 adopts strategy D in equilibrium, as Proposition 1 suggests.

Next, rearranging Firm 1 profit, \( \pi_1 \), of Equation (13) by using Equations (14) and (15) gives \( \pi_1 = q_1^2 \), which is convex with respect to Firm 1's quantity. Note that Firm 1 wishes to decrease the quantity supplied by Firm 0. Because the objective function is convex, I consider the situation where Firm 1 obtains and discloses 'good' private information; that is, the marginal cost for Firm 1 is low. On learning this information, Firm 0 reduces its supply for the purpose of social welfare maximization for the following reason. A low marginal cost for Firm 1 means that Firm 1 can produce goods more efficiently thanks to lower production costs. In that case, it is better for Firm 0 to reduce its own supply and let Firm 1 increase its supply, because such behavior lowers total production costs for the two firms, leading to positive impacts on social welfare. Moreover, observe that Equation (17) suggests that Firm 0

\(^{14}\) 'Strategic substitute' in Cournot competition means that if one firm commits to decrease (increase) its supply, the other firm will strategically increase (decrease) its supply to increase its profit in response. See Bulow et al. (1985).
never pursues its own profit. Hence, Firm 0 decreases its supply, which in turn results in a higher profit for Firm 1 compared with the situation where Firm 1 would not have disclosed the good news. This is the reason that Firm 1 adopts strategy D in the mixed duopoly. In other words, the SOE wishes most of the demand to be fulfilled by the firm with the lowest marginal production costs. Firm 1 knows this and thus benefits from disclosing good news because the SOE will transfer part of its demand to Firm 1 by decreasing its quantity.

4.2.2. Bertrand-type Price Competition Case

Substituting $e = 0$ into Equations (10) and (11) gives:

$$SS = \left( p_0 - p_1 \right) (2(k_0 - k_1) - p_0 + p_1 + 2(1-\theta)(a - 2c - k_0 - k_1) + c(p_0 + p_1) + k_i p_0 + k_0 p_1 - p_0 p_1) \right) / 2(1-\theta)(1 + \theta),$$

(18)

$$\pi_1 = \left( p_1 - c - k_1 \right) (1 - \theta) a - p_1 + \theta p_0) / (1 - \theta)(1 + \theta).$$

(19)

Firm 0 maximizes Equation (18) with respect to $p_0$, and Firm 1 maximizes Equation (19) with respect to $p_1$. In the absence of uncertainty, solving $\partial SS / \partial p_0 = \partial \pi_1 / \partial p_1 = 0$ for $p_0$ and $p_1$ yields:

$$p_0 = \left( 1 - \theta \right) a + \left( 2 - \theta \right) c + 2k_0 - \theta k_1 \right) / (2 - \theta^2),$$

(20)

$$p_1 = \left( 1 - \theta \right) a + \left( 1 + \theta - \theta^2 \right) c + \theta k_0 + \left( 1 - \theta^2 \right) k_1 \right) / (2 - \theta^2).$$

(21)

First, consider the disclosure strategy undertaken by Firm 0. Rearranging Equation (18) by using the equilibrium prices of Equations (20) and (21), I derive

$$SS = -(p_0 - c - k_0)^2 / (2\theta^2) + (a - c)(a - c - k_0 - k_1) / (1 + \theta) + \left( k_0^2 + k_1^2 - 2\theta k_0 k_1 \right) / (2(1 - \theta^2)),

which is concave with respect to $p_0$. Because the objective for Firm 0 is concave, consider the situation where Firm 0 receives 'bad' information that its marginal cost is high and discloses it publicly as in the Cournot case. Learning that the marginal cost for Firm 0 is high, Firm 1 will raise its selling price to boost its own profit, because price works as a strategic complement in Bertrand competition. Firm 1's higher price also means that the supply from Firm 1 decreases. Since the resulting decrease in equilibrium total demand has a negative impact on consumer surplus, Firm 0 does not disclose bad information. In other words, in Bertrand competition, Firm 1 will use any information to improve its own profit at the expense of consumer surplus. Hence, there is no benefit to Firm 0 to provide Firm 1 with more information. Consequently, nondisclosure is the optimal strategy for Firm 0.

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15 'Strategic complement' in Bertrand competition means that if one firm commits to increase (decrease) the price, the other firm strategically increases (decreases) the price to increase its profit in response. See Bulow et al. (1985).
On the other hand, rearranging Firm 1 profit, \( \pi_1 \), of Equation (19) by using Equations (20) and (21) gives
\[
\pi_1 = \frac{(p_1 - c - k_1)^2}{(1 - \theta^2)}.
\]
which is convex with respect to \( p_1 \). Note that Firm 1 would like to increase the price set by Firm 0. Because of the convexity, suppose that Firm 1 obtains and discloses 'good' information that its marginal cost is low. Equation (A16) in the appendix shows that
\[
\hat{p}_0 = \theta E(p_1 - k_1 | k_0, \kappa_0, \kappa_1) + (1 - \theta)c + k_0
\]
is the optimal response for Firm 0. This equation shows that if \( k_1 \) takes a lower value, Firm 0 will raise the price, \( p_0 \), ceteris paribus. The reason why a decrease in the marginal cost of Firm 1 increases the price of Firm 0, which is different from a normal profit-maximizing duopoly, is basically the same as in the Cournot case. Firm 0 wishes most of the demand to be fulfilled by the firm with the lowest marginal costs of production. Hence, by raising its price, it transfers demand from Firm 0 to Firm 1, thereby increasing consumer surplus. As a consequence, disclosure is optimal for Firm 1.

5. Model with Demand Uncertainty

5.1. Equilibrium Results

This section shifts my focus from cost uncertainty to demand uncertainty. Specifically, while I again assume that \( e \), which was assumed to be zero in the previous section, is defined as a random variable, \( k_i \) is now assumed to be deterministically zero throughout this section. Taking steps similar to those in the previous section, I identify the Bayesian equilibrium strategies and payoffs by the type of competition mode as summarized in the following lemmas.

Lemma 3 The Bayesian equilibrium strategy under Cournot competition with demand uncertainty features the following quantity choices:
\[
\hat{q}_0 = \frac{2 - \theta(a - c)}{(2 - \theta^2)} + W_0^C f_0 + Y_0^C \rho_0 + Z_0^C \rho_1,
\]
\[
\hat{q}_1 = \frac{1 - \theta(a - c)}{(2 - \theta^2)} + W_1^C f_1 + Y_1^C \rho_0 + Z_1^C \rho_1,
\]
where
\[
W_0^C = \frac{V(2(\Delta_1 + \sigma_1)(\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1)) - \theta V(\Delta_1(\Delta_0 + \sigma_0))}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}
\]
\[
Y_0^C = \frac{\theta V(2 - \theta)\sigma_0(\Delta_0 + \sigma_0 + \sigma_1)(\sigma_0(\Delta_1 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta(1 - \theta)W_0^C \sigma_1}{2 - \theta^2}) \frac{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}{2 - \theta^2}
\]
\[
Z_0^C = \frac{\theta V(2 - \theta)\sigma_0(\Delta_0 + \sigma_0 + \sigma_1)(\sigma_0(\Delta_1 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta(1 - \theta)W_0^C \sigma_1}{2 - \theta^2}) \frac{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}{2 - \theta^2}
\]
and
The corresponding expected payoffs equal:

$$W_1^c = \frac{V(\sigma_1(\Delta_1 + \sigma_0) + V(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1)) + \Delta_0((1 - \theta)(\Delta_1 + \sigma_0 + \sigma_0)))}{2((\Delta_0 + \sigma_0)(\sigma_1 + V(\Delta_1 + \sigma_0 + \sigma_1)) + V(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + (1 - \theta)(\Delta_1 + \sigma_0 + \sigma_0)))} - \theta^2 \Delta_0 \Delta_1 V^2$$

$$Y_1^c = \frac{V(2(1 - \theta)(\sigma_1(\Delta_1 + \sigma_0) + V(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1) - \theta^2 \Delta_0 \Delta_1 V^2)}{2((\Delta_0 + \sigma_0)(\sigma_1 + V(\Delta_1 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2)}$$

$$Z_1^c = \frac{\theta V(\sigma_1(\Delta_1 + \sigma_0) + V(\Delta_1 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2)}{2((\Delta_0 + \sigma_0)(\sigma_1 + V(\Delta_1 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2).}$$

The corresponding expected payoffs equal:

$$E(SS(\hat{q}_0, \hat{q}_1)) = \left(7 - 6\theta - 2\theta^2 + 2\theta^3\right)(a - c)^2 / \left(2(1 - \theta)^2\right) + V(\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2$$

$$- V\left((\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1))\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)\right)^2 \sigma_0 + \left((\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1))\sigma_0 + V(\Delta_1 + \sigma_0 + \sigma_1)\right)^2 \sigma_0 / 2$$

$$- \theta V\left((\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1))\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)\right) + \theta V\left((\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1))\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)\right)^2 \sigma_0$$

$$E(\pi_1(\hat{q}_0, \hat{q}_1)) = (1 - \theta)^2(a - c)^2 / \left(2(1 - \theta)^2\right) + V(\sigma_1(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2.$$

**Lemma 4** The Bayesian equilibrium strategy under Bertrand competition with demand uncertainty features the following price choices:

$$\hat{p}_0 = (1 - \theta)\alpha + (2 - \theta)\beta / (2 - \theta^2) + W_0^B f_0 + Y_0^B \rho_0 + Z_0^B \rho_1,$$

$$\hat{p}_1 = (1 - \theta)\alpha + (1 + \theta - \theta^2)\beta / (2 - \theta^2) + W_1^B f_1 + Y_1^B \rho_0 + Z_1^B \rho_1,$$

where

$$W_0^B = \frac{\theta (1 - \theta)^2 \Delta_1 (\Delta_1 + \sigma_0)}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}$$

$$Y_0^B = \frac{\theta (1 - \theta)^2 \Delta_1 (\Delta_1 + \sigma_0)}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}$$

$$Z_0^B = \frac{\theta (1 - \theta)^2 \Delta_1 (\Delta_1 + \sigma_0)}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}$$

and

$$W_1^B = \frac{(1 - \theta)\beta (\Delta_0 + \sigma_0)(\sigma_1(\Delta_1 + \sigma_0) + V(\Delta_1 + \sigma_0 + \sigma_1))}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}$$

$$Y_1^B = \frac{(1 - \theta)\beta (\Delta_0 + \sigma_0)(\sigma_1(\Delta_1 + \sigma_0) + V(\Delta_1 + \sigma_0 + \sigma_1))}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}$$

$$Z_1^B = \frac{\theta (1 - \theta)\beta (\Delta_0 + \sigma_0)(\sigma_1(\Delta_1 + \sigma_0) + V(\Delta_1 + \sigma_0 + \sigma_1))}{2((\Delta_0 + \sigma_0)\sigma_1 + V(\Delta_0 + \sigma_0 + \sigma_1))(\sigma_0(\Delta_1 + \sigma_0 + \sigma_1) + V(\Delta_1 + \sigma_0 + \sigma_1)) - \theta^2 \Delta_0 \Delta_1 V^2}.$$
The following lemmas follow from Lemmas 3 and 4.

**Lemma 5** In Cournot competition with demand uncertainty, if the joint-stock company discloses its demand information, disclosure is the optimal response for the SOE. Further, if the SOE discloses its demand information, no disclosure is the optimal response for the joint-stock company.

**Lemma 6** In Bertrand competition with demand uncertainty, disclosure is the dominant strategy for the joint-stock company. If the joint-stock company discloses its private signal, no disclosure is the optimal response for the SOE.

Using Lemmas 5 and 6, I identify the following equilibria.

**Proposition 3** Under Cournot competition with demand uncertainty, the equilibrium strategies are as follows: (1) the SOE discloses market information while the joint-stock company does not disclose information if \( \sigma_1 \leq \Sigma \) or \( \sigma_1 \geq (2-\theta)\theta V \sigma_0 / (2(1-\theta)(V+\sigma_0)) \), and (2) neither firm discloses information if \( \Sigma \leq \sigma_1 \leq (2-\theta)\theta V \sigma_0 / (2(1-\theta)(V+\sigma_0)) \). \( \Sigma \) is defined as:

\[
\Sigma = \frac{(V((2-\theta^2)(V - 2(6 - 12\theta + 3\theta^2 + 4\theta - 2\theta^2)\sigma_0 + 4(3 - 2\theta^2)\sigma_0^2 - (1 - \theta)(6 - 12\theta + 3\theta^2 + 4\theta^3 - 2\theta^4)\sigma_0))) / (8(1-\theta)(3-\theta^2)(V+\sigma_0))}{(V - 2(6 - 12\theta + 3\theta^2 + 4\theta^3 - 2\theta^4)\sigma_0)^2}.
\]

**Proposition 4** Under Bertrand competition with demand uncertainty, the equilibrium is that the SOE does not disclose market information, while the joint-stock company discloses information.

Propositions 3 and 4 show that the equilibrium disclosure policy under Cournot competition is more complicated than under Bertrand competition. While the inequality identifying the optimal disclosure policy under Cournot competition is stated in terms of \( \sigma_1 \).
(e.g., $\Sigma \leq \sigma_i \leq (2 - \theta)\theta W \sigma_0 / (2(1 - \theta)(V + \sigma_0))$, it would be easier to compare the results from the mixed duopoly with the results in a profit-maximizing duopoly if the inequality were formulated in terms of $\theta$. However, it is impossible to yield explicit solutions of $\theta$ by solving this inequality, because $\Sigma$ is a complex function that includes $\theta$.

5.2. Rationale

5.2.1. Cournot-type Quantity Competition Case

First, consider the case of Cournot competition. It should be noted that Suijs and Wielhouwer (2014) demonstrate that whether an uncertain factor correlates or not between two firms matters in Cournot competition for equilibrium disclosure policy, while the type of uncertainty (i.e., cost or demand) does not. Their insight suggests that the interpretation of Cournot competition with demand uncertainty can basically be based upon that under cost uncertainty. Indeed, objective functions of Equations (8) and (9) suggest that it is irrelevant for equilibrium whether uncertainty is associated with demand or cost, because only the linear difference between $e$ and $k_i$ (i.e., $e - k_i$) is included as the random factor in the objective functions.

Concentrating on demand uncertainty, I substitute $k_0 = k_1 = 0$ into the objective functions of Equations (8) and (9), yielding:

$$SS = (a + e - c)(q_0 + q_1) - \left(q_0^2 + q_1^2 + 2\theta q_0 q_1\right)/2,$$
$$\pi_1 = (a + e - (q_1 + \theta q_0) - c)q_1.$$  (22)

Firm 0 maximizes $SS$ with respect to $q_0$, and Firm 1 maximizes $\pi_1$ with respect to $q_1$ if both firms face no uncertainty. That is, solving $\partial SS / \partial q_0 = \partial \pi_1 / \partial q_1 = 0$ for $q_0$ and $q_1$ gives:

$$q_0 = (2 - \theta)(a - c + e)/(2 - \theta^2),$$
$$q_1 = (1 - \theta)(a - c + e)/(2 - \theta^2).$$  (24)

Rearranging Equation (22) using Equation (24) restates $SS$ as:

$$SS = -(1 - \theta^2)\left(q_0 - (a + e - c)/(1 + \theta)\right)^2 / (2\theta^2) + (a + e - c)^2 / (1 + \theta),$$

which is concave with respect to $q_0$. Recall that Firm 0’s objective is to maximize total demand $q_0 + q_1$. Suppose that Firm 0 possesses and discloses ‘bad’ information that demand is low. Then, the strategic information effect causes Firm 1 to increase its quantity similar to the cost uncertainty case. However, the market information effect in addition to the strategic information effect also arises in the demand uncertainty case, as discussed at the end of
Section 3. Due to the market information effect, Firm 1 also reduces its quantity if it receives market information that demand is low, which in turn leads to a reduction in consumer surplus. Therefore, the optimal disclosure strategy for Firm 0 depends upon which of the two effects dominates. If the strategic information effect dominates, strategy D is the optimal strategy for Firm 0. By contrast, if the market information effect dominates, strategy ND is optimal. Remember that Strategy D is optimal under cost uncertainty in Cournot competition because only the strategic information effect arises. Under demand uncertainty, however, the market information effect may dominate, and therefore nondisclosure can be the optimal strategy. Note that the magnitude of the strategic and market information effects depends on the precision of the firms' private information; that is, the variance parameters, $\sigma_i$. The more precise the private information, the more strongly a firm responds to disclosure of this information.

With respect to Firm 1 profits, rearranging Equation (23) by using Equation (24), I derive $\pi_1 = q_1^2$, which is convex with respect to $q_1$. Note that Firm 1 wants to decrease the quantity supplied by Firm 0. Suppose that Firm 1 obtains and discloses 'good' information that there is a relatively high demand. Because Firm 1 increases supply, Firm 0 may also increase supply to further reduce price and to increase total demand, and thus consumer surplus owing to the strategic information effect. Meanwhile, the market information effect will also increase the supply of Firm 0 because Firm 1 only serves a part of the increased demand; that is, Firm 1 will increase its supply by a lower amount than the positive demand shock. Firm 0 is also willing to fulfill this demand and therefore increases its supply as well. In summary, disclosure of good demand information will increase the supply by Firm 0 and thus decrease price. Firm 1 does not like that, and so it prefers to not disclose. Based on the eventual outcome, the market information effect dominates the strategic information effect.

5.2.2. Bertrand-type Price Competition Case

Finally, I examine Bertrand competition with demand uncertainty. Substitution of $k_0 = k_1 = 0$ into Equations (10) and (11) gives:

$$SS = \frac{1}{2}(1 - \theta)(2(a + e)(a + e - 2c) + 2(c(p_0 + p_1) - p_0^2 - p_1^2 + 2\theta p_0 p_1) + (2(1 - \theta)(1 + \theta))}, \quad (26)$$
$$\pi_1 = \frac{(p_1 - c)((1 - \theta)(a + e) - p_1 + \theta p_0)}{(1 - \theta)(1 + \theta)}. \quad (27)$$

Firm 0 maximizes $SS$ with respect to $p_0$ and Firm 1 maximizes $\pi_1$ with respect to $p_1$ in the absence of uncertainty. Solving $\partial SS/\partial p_0 = \partial \pi_1/\partial p_1 = 0$ for $p_0$ and $p_1$ gives:

$$p_0 = \frac{\theta(1 - \theta)(a + e) + (2 - \theta) c}{(2 - \theta^2)}, \quad (28)$$
$$p_1 = \frac{(1 - \theta)(a + e) + (1 + \theta - \theta^2) c}{(2 - \theta^2)}.$$
Rearranging Equation (26) using Equation (28), I derive
\[ SS = -(p_o - c)^2/((2\theta^2) + (a + e - c)^2)/(1 + \theta), \]
which is concave and decreasing with respect to Firm 0's net price. Here, suppose that Firm 0 obtains and discloses good information that demand is high. Then, any information about demand that Firm 0 provides to Firm 1 will be exploited and will reduce consumer surplus. Specifically, if Firm 0 discloses good news, Firm 1 will increase its price due to the market information effect, which reduces consumer surplus. Hence, Firm 0 is better off by not disclosing the good news.

Meanwhile, rearranging Equation (27) using Equation (28) gives
\[ \pi_1 = (p_1 - c)^2/((1 - \theta)(1 + \theta)), \]
which is convex and increasing with respect to Firm 1's net price. Note that Firm 1 wishes to maximize Firm 0's price. Because Firm 1's profit is convex with respect to net price, suppose that Firm 1 discloses 'good' information that demand is high. Following Equation (A42),
\[ \hat{p}_0 = \theta E(\hat{p}_1 | f_o, \rho_o, \rho_1) + (1 - \theta)c, \]
Firm 0 raises its price, because Firm 0 also cares about the firms' profits. Eventually, Firm 1 benefits from disclosing the good demand news.

6. Conclusions

This paper investigates the optimal disclosure policy for private information under a mixed duopoly environment, a topic hitherto unaddressed in either the accounting or economic literature. Many existing studies provide results concerning the manner in which disclosure policy constitutes an equilibrium strategy when competition arises between profit-maximizing firms. Contrary to these conventional insights regarding profit-maximizing firm competition, this paper has demonstrated that an asymmetric equilibrium strategy between firms can instead arise in a mixed duopoly. Table 1 summarizes the combination of the equilibrium disclosure strategies taken by the SOE and the joint-stock company by type of competition and uncertainty.

A notable result in Table 1 is that the SOE does not disclose its information in Bertrand competition under either uncertainty. The explanation for this stance is that the joint-stock company will exploit any information that the SOE provides. That is, Firm 1 will raise the equilibrium price on average after receiving additional information in Bertrand competition. This price increase reduces total supply quantity and consumer surplus, which is not in line with the SOE's objective.

One practical implication of this study is that the full disclosure of information by the SOE does not necessarily enhance social welfare. Intuitively, the SOE should fully disclose its information on market conditions it has received if it pursues the public interest, but this is not necessarily the case under certain competitive environments. This finding yields new insights into public sector accounting in that an SOE should exercise caution in determining whether to disclose its information following careful examination of how a joint-stock
company in the same market will strategically react to disclosure as well as to the market environment in which it operates. In particular, when an SOE and a joint-stock company face Bertrand-type price competition, the same disclosure rules should not apply to the SOE and the joint-stock company. However, the overview in Section 2 suggests that the same level of disclosure standards has been applied recently to SOEs as to listed joint-stock companies. The results in this paper pose an important question about the trend in disclosure regulations for SOEs.

Appendix

Proof of Lemma 1. I first derive the BNE strategies adopted by the two firms at Date 4. Based on Equations (8) and (9), the expected payoffs conditional on available information for the two firms are stated as follows:

\[ E(SS \mid k_0, \kappa_0, \kappa_1) = a(q_0 + E(q_1 \mid k_0, \kappa_0, \kappa_1)) \]
\[ -\frac{1}{2}(q_0^2 + E(q_1^2 \mid k_0, \kappa_0, \kappa_1)) + 2\theta q_0 E(q_1 \mid k_0, \kappa_0, \kappa_1) \]
\[ - (c + k_0)q_0 - E((c + k_1)q_1 \mid k_0, \kappa_0, \kappa_1) \]

(A1)

\[ E(\pi_1 \mid k_1, \kappa_0, \kappa_1) = (a - c - k_1 - q_1 - \theta E(q_0 \mid k_1, \kappa_0, \kappa_1))q_1 . \]

(A2)

The first-order derivatives of the expected payoffs are respectively stated as:

\[ \frac{\partial E(SS \mid k_0, \kappa_0, \kappa_1)}{\partial q_0} = a - c - k_0 - q_0 - \theta E(q_1 \mid k_0, \kappa_0, \kappa_1) \]
\[ \frac{\partial E(\pi_1 \mid k_1, \kappa_0, \kappa_1)}{\partial q_1} = a - c - k_1 - 2q_1 - \theta E(q_0 \mid k_1, \kappa_0, \kappa_1) . \]

By solving each of the two derivatives as equal to zero, I derive the following expected reaction functions under uncertainty:

\[ \hat{q}_0 = a - c - k_0 - \theta E(q_1 \mid k_0, \kappa_0, \kappa_1) , \hat{q}_1 = (a - c - k_1 - \theta E(q_0 \mid k_1, \kappa_0, \kappa_1))/2 . \]

(A3)

Using these reaction functions under uncertainty, I next conjecture that quantity choices are linear in the information variables; that is, \( \hat{q}_i = X_i + Y_i \kappa_0 + Z_i \kappa_1 + W_i k_i \) for each firm \( i = 0, 1 \).

Substituting these linear conjectures of \( \hat{q}_0 \) and \( \hat{q}_1 \) into Equation (A3) gives:

\[ X_0 + Y_0 \kappa_0 + Z_0 \kappa_1 + W_0 k_0 = a - c - k_0 - \theta(X_1 + Y_1 \kappa_0 + Z_1 \kappa_1 + W_1 E(k_1 \mid k_0, \kappa_0, \kappa_1)) , \]

(A4)

\[ X_1 + Y_1 \kappa_0 + Z_1 \kappa_1 + W_1 k_1 = (a - c - k_1 - \theta(X_0 + Y_0 \kappa_0 + Z_0 \kappa_1 + W_0 E(k_0 \mid k_1, \kappa_0, \kappa_1)))/2 . \]

(A5)
Combining Equations (A4) and (A5) and using $E(k_i | k_j, \kappa_0, \kappa_1) = (s_i / (s_i + \Delta_i))k_i$ for $(i, j) = (0, 1)$ or $(1, 0)$ yields the following series of equations.

\[ X_0 = a - c - \theta X_1 \quad \text{(A6)} \]

\[ X_1 = (a - c - \theta X_0) / 2 \quad \text{(A7)} \]

\[ Y_0 = -\theta Y_1 \quad \text{(A8)} \]

\[ Y_1 = -\theta(Y_0 + s_0 W_0 / (s_0 + \Delta_0)) / 2 \quad \text{(A9)} \]

\[ Z_0 = -\theta(Z_1 + s_1 W_1 / (s_1 + \Delta_1)) \quad \text{(A10)} \]

\[ Z_1 = -\theta Z_0 / 2 \quad \text{(A11)} \]

\[ W_0 = -1 \quad \text{(A12)} \]

\[ W_1 = -1 / 2 \quad \text{(A13)} \]

Combining Equations (A6)–(A13) to solve for $(X_0, X_1, Y_0, Y_1, Z_0, Z_1, W_0, W_1)$ and substituting the solutions into $q_i = X_i + Y_i \kappa_0 + Z_i \kappa_1 + W_i k_i$ $(i = 0, 1)$ yields the equilibrium strategies $(\hat{q}_0, \hat{q}_1)$ as shown in this lemma.

Finally, I substitute $(\hat{q}_0, \hat{q}_1)$ and $\kappa_i = k_i + \delta_i$ $(i = 0, 1)$ into Equations (A1) and (A2) to yield the unconditional expected payoffs at Date 1. Using $E(k_i^2) = s_i$, $E(\delta_i^2) = \Delta_i$ and $E(k_i) = E(\delta_i) = 0$ $(i = 0, 1)$, I derive E(SS) and E(\pi) as shown in this lemma.

**Proof of Lemma 2.** Based on Equations (10) and (11), the expected payoffs conditional on available information for the two firms at Date 4 are written as:

\[ E(SS | k_0, \kappa_0, \kappa_1) = (2(1 - \theta)(a(a - 2c - k_0) + cp_0) + 2k_0 p_0 - p_0^2 - p_1^2 - 2((1 - \theta)a + \theta p_0)E(k_i | k_0, \kappa_0, \kappa_1) + 2((1 - \theta)c + \theta(p_0 - k_0))E(p_i | k_0, \kappa_0, \kappa_1) + 2E(k_i p_i | k_0, \kappa_0, \kappa_1) / (2(1 - \theta^2)) \]

\[ E(\pi_i | k_1, \kappa_0, \kappa_1) = (p_i - c - k_1)((1 - \theta)a - p_i + \theta E(p_0 | k_1, \kappa_0, \kappa_1)) / (1 - \theta^2) \]

The first-order derivatives of the expected payoffs are respectively stated as follows:
\[
\frac{\partial E(S_i | k_0, \kappa_0, \kappa_1)}{\partial p_0} = -p_0 + \theta E(p_1 | k_0, \kappa_0, \kappa_1) + (1 - \theta) c + k_0 - \theta E(k_1 | k_0, \kappa_0, \kappa_1) \frac{1}{1 - \theta^2},
\]

\[
\frac{\partial E(\pi_i | k_1, \kappa_0, \kappa_1)}{\partial p_1} = (1 - \theta) a + c + \theta E(p_0 | k_1, \kappa_0, \kappa_1) - 2 p_1 + k_1 \frac{1}{1 - \theta^2}.
\]

By solving the two derivatives as equal to zero, I derive the following reaction functions under uncertainty.

\[\hat{p}_0 = \theta E(\hat{p}_1 - k_1 | k_0, \kappa_0, \kappa_1) + (1 - \theta) c + k_0, \quad \hat{p}_1 = (\theta E(\hat{p}_0 | k_1, \kappa_0, \kappa_1) + (1 - \theta) a + c + k_1) / 2 \quad (A16)\]

Using Equation (A16), I next conjecture that price choices are linear in the information variables, that is, \[\hat{p}_i = X_i + Y_i\kappa_0 + Z_ik_i + W_ik_i \quad \text{for each firm } i = 0, 1.\] Substituting these linear conjectures of \(\hat{p}_0\) and \(\hat{p}_1\) into Equation (A16) gives:

\[X_0 + Y_0\kappa_0 + Z_0\kappa_1 + W_0k_0 = \theta(X_1 + Y_1\kappa_0 + Z_1\kappa_1 + (W_1 - 1)E(k_1 | k_0, \kappa_0, \kappa_1) + (1 - \theta) c + k_0, \quad (A17)\]

\[X_1 + Y_1\kappa_0 + Z_1\kappa_1 + W_1k_1 = (\theta(X_0 + Y_0\kappa_0 + Z_0\kappa_1 + W_0E(k_0 | k_1, \kappa_0, \kappa_1)) + (1 - \theta) a + c + k_1) / 2 . \quad (A18)\]

Combining Equations (A17) and (A18) and using \(E(k_i | k_j, \kappa_0, \kappa_1) = (s_j / (s_j + \Delta_j))\kappa_i \quad \text{for } (i, j) = (0, 1) \text{ or } (1, 0)\) yields the following equations.

\[X_0 = \theta X_1 + (1 - \theta) c \quad \text{(A19)}\]
\[X_1 = ((1 - \theta) a + c + \theta X_0) / 2 \quad \text{(A20)}\]
\[Y_0 = \theta Y_1 \quad \text{(A21)}\]
\[Y_1 = \theta(Y_0 + s_0W_0 / (s_0 + \Delta_0)) / 2 \quad \text{(A22)}\]
\[Z_0 = \theta(Z_1 + s_1(W_1 - 1) / (s_1 + \Delta_1)) \quad \text{(A23)}\]
\[Z_1 = \theta Z_0 / 2 \quad \text{(A24)}\]
\[W_0 = 1 \quad \text{(A25)}\]
\[W_1 = 1 / 2 \quad \text{(A26)}\]

Combining Equations (A19)–(A26) to solve for \((X_0, X_1, Y_0, Y_1, Z_0, Z_1, W_0, W_1)\) and substituting the solutions into \(\hat{p}_i = X_i + Y_i\kappa_0 + Z_i\kappa_1 + W_ik_i \quad (i = 0, 1)\) yields the equilibrium strategies.
$(\hat{p}_0, \hat{p}_1)$ as shown in this lemma.

Finally, I substitute $(\hat{p}_0, \hat{p}_1)$ and $\kappa_i = k_i + \delta_i$ ($i = 0, 1$) into Equations (A14) and (A15) to yield the unconditional expected payoffs at Date 1. Using $E(k_i^2) = \sigma_i$, $E(\delta_i^2) = \Delta_i$ and $E(k_i) = E(\delta_i) = 0$ ($i = 0, 1$), I derive $E(SS)$ and $E(\pi_1)$ as shown in this lemma.

**Proof of Proposition 1.** I hereafter use a superscript to denote the combinations of equilibrium strategies throughout this appendix. The first letter in parenthesis denotes the strategy of Firm 0, whereas the second denotes the strategy of Firm 1 at Date 1. For example, superscript (D, ND) signifies the outcome where Firm 0 discloses information but Firm 1 does not. From Lemma 1, the following inequalities hold.

$$E(SS^{(D,D)}) - E(SS^{(ND,ND)}) = E(SS^{(D,ND)}) - E(SS^{(ND,D)}) = \theta^2(3 - \theta^2)k_0 / \left(2(2 - \theta^2)^2\right) > 0$$

$$E(\pi_1^{(D,D)}) - E(\pi_1^{(ND,ND)}) = E(\pi_1^{(D,ND)}) - E(\pi_1^{(ND,D)}) = \theta^2(4 - \theta^2)k_1 / \left(4(2 - \theta^2)^2\right) > 0$$

The inequalities indicate that D is the dominant strategy for both firms.

**Proof of Proposition 2.** From Lemma 2, the following inequalities hold.

$$E(SS^{(ND,D)}) - E(SS^{(D,D)}) = E(SS^{(ND,ND)}) - E(SS^{(D,ND)}) = \theta^2k_0 / \left(2(2 - \theta^2)^2\right) > 0$$

$$E(\pi_1^{(D,D)}) - E(\pi_1^{(ND,ND)}) = E(\pi_1^{(D,ND)}) - E(\pi_1^{(ND,D)}) = \theta^2(4 - \theta^2)k_1 / \left(4(1 - \theta)(1 + \theta)(2 - \theta^2)^2\right) > 0$$

The inequalities indicate that ND is the dominant strategy for Firm 0 while D is the dominant strategy for Firm 1.

**Proof of Lemma 3.** I first derive the BNE strategies chosen by the two firms at Date 4. Based on Equations (8) and (9), the expected payoffs conditional on available information for the two firms are stated as follows:

$$E(SS \mid f_0, \rho_0, \rho_1) = (a + E(e \mid f_0, \rho_0, \rho_1) - c)q_0 + E(q_1 \mid f_0, \rho_0, \rho_1) + \frac{1}{2}q_0^2 + E(q_1^2 \mid f_0, \rho_0, \rho_1) + 2\theta q_0 E(q_1 \mid f_0, \rho_0, \rho_1), (A27)$$

$$E(\pi_1 \mid f_1, \rho_0, \rho_1) = (a + E(e \mid f_1, \rho_0, \rho_1) - c - q_1 - \theta E(q_0 \mid f_1, \rho_0, \rho_1))q_1. (A28)$$

The first-order derivatives of the expected payoffs are respectively stated as:
\[
\frac{\partial E(SS | f_0, \rho_0, \rho_1)}{\partial q_0} = a + E(e | f_0, \rho_0, \rho_1) - c - q_0 - \theta E(q_1 | f_0, \rho_0, \rho_1),
\]

\[
\frac{\partial E(\pi_1 | f_1, \rho_0, \rho_1)}{\partial q_1} = a + E(e | f_1, \rho_0, \rho_1) - c - 2q_1 - \theta E(q_0 | f_1, \rho_0, \rho_1).
\]

By solving each of the two derivatives as equal to zero, I derive the following expected reaction functions under uncertainty.

\[
\hat{q}_0 = a + E(e | f_0, \rho_0, \rho_1) - c - \theta E(\hat{q}_1 | f_0, \rho_0, \rho_1),
\]

\[
\hat{q}_1 = \left( a + E(e | f_1, \rho_0, \rho_1) - c - \theta E(\hat{q}_0 | f_1, \rho_0, \rho_1) \right) / 2 \tag{A29}
\]

Using Equation (A29), I conjecture that the quantity choices are linear in the information variables, that is, \( \hat{q}_i = X_i^C + Y_i^C \rho_0 + Z_i^C \rho_1 + W_i^C f_i \) for each firm \( i = 0, 1 \). Substituting these linear conjectures of \( \hat{q}_0 \) and \( \hat{q}_1 \) into Equation (A29) gives:

\[
X_0^C + Y_0^C \rho_0 + Z_0^C \rho_1 + W_0^C f_0 = a + E(e | f_0, \rho_0, \rho_1) - c - \theta \left( X_1^C + Y_1^C \rho_0 + Z_1^C \rho_1 + W_1^C e \left( f_1 | f_0, \rho_0, \rho_1 \right) \right), \tag{A30}
\]

\[
X_1^C + Y_1^C \rho_0 + Z_1^C \rho_1 + W_1^C f_1 = \left( a + E(e | f_1, \rho_0, \rho_1) - c - \theta \left( X_0^C + Y_0^C \rho_0 + Z_0^C \rho_1 + W_0^C e \left( f_0 | f_1, \rho_0, \rho_1 \right) \right) \right) / 2. \tag{A31}
\]

Combining Equations (A30) and (A31) and using the fact that \( E(e | f_i, \rho_0, \rho_1) = V \left( (\sigma_j + \Delta_j) f_i + \sigma_i \rho_j \right) \left( (\sigma_i + \sigma_j + \Delta_j) \right) + V(\sigma_i + \sigma_j + \Delta_j) \) and \( E(f_i | f_j, \rho_0, \rho_1) = V \left( \Delta_j f_i + \sigma_i \sigma_j + V(\sigma_i + \sigma_j) \right) \left( (\sigma_i + \sigma_j + \Delta_j) \right) \) for \( (i, j) = (0, 1) \) or \( (1, 0) \) (DeGroot, 1970: 55) yields the following equations.

\[
X_0^C = a - c - \theta X_1^C \tag{A32}
\]

\[
X_1^C = \left( a - c - \theta X_0^C \right) / 2 \tag{A33}
\]

\[
Y_0^C = -\theta Y_1^C \tag{A34}
\]

\[
Y_1^C = -\left( \left( \theta(\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))W_0^C - V\sigma_1 \right) \left( (\sigma_0 + \sigma_1 + \Delta_0) + V(\sigma_0 + \sigma_1 + \Delta_0) \right) + \theta Y_0^C \right) / 2 \tag{A35}
\]

\[
Z_0^C = -\left( \left( \theta(\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))W_0^C - V\sigma_0 \right) \left( (\sigma_0 + \sigma_1 + \Delta_1) + V(\sigma_0 + \sigma_1 + \Delta_1) \right) + \theta Z_1^C \right) \tag{A36}
\]

\[
Z_1^C = -\theta Z_0^C / 2 \tag{A37}
\]
\[ W_0^C = V(\sigma_i + \Delta_i - \theta \Delta_i W_1^C)/(\sigma_0(\sigma_i + \Delta_i) + V(\sigma_0 + \sigma_i + \Delta_i)) \]  
(A38)
\[ W_1^C = V(\sigma_0 + \Delta_0 - \theta \Delta_0 W_0^C)/(2(\sigma_0 + \Delta_0)\sigma_1 + V(\sigma_0 + \sigma_1 + \Delta_0)) \]  
(A39)

Combining Equations (A32)–(A39) to solve for \((X_0^C, X_1^C, Y_0^C, Y_1^C, Z_0^C, Z_1^C, W_0^C, W_1^C)\) and substituting the solutions into \(\hat{q}_i = X_i^C + Y_i^C \rho_0 + Z_i^C \rho_1 + W_i^C f_i \) \((i = 0, 1)\) yields the equilibrium strategies \((\hat{q}_0, \hat{q}_1)\) as shown in this lemma.

Finally, I substitute \((\hat{q}_0, \hat{q}_1), f_i = e + \varepsilon, \) and \(\rho_i = e + \varepsilon + \delta_i \) \((i = 0, 1)\) into Equations (A27) and (A28) to yield the unconditional expected payoffs at Date 1. Using \(E(e^2) = V,\) \(E(\varepsilon_i^2) = \sigma_i, \ E(\delta_i^2) = \Delta_i, \) and \(E(e) = E(\varepsilon_i) = E(\delta_i) = 0 \) \((i = 0, 1)\), I derive \(E(SS)\) and \(E(\pi_i)\) in equilibrium as shown in this lemma. □

**Proof of Lemma 4.** I first derive the BNE strategies. Based on Equations (10) and (11), the expected payoffs conditional on available information for the two firms are stated as follows:
\[
E(SS \mid f_0, \rho_0, \rho_1) = (2(1-\theta)(a(a-2c)+cp_0) - p_0^2 - E(p_1^2 \mid f_0, \rho_0, \rho_1) \]
\[+ 2((1-\theta)c + \phi_0)E(p_1 \mid f_0, \rho_0, \rho_1) + 4(1-\theta)(a-c)E(e \mid f_0, \rho_0, \rho_1) \]
\[+ 2(1-\theta)E(e^2 \mid f_0, \rho_0, \rho_1))/(2(1-\theta^2)), \]
(A40)
\[
E(\pi_i \mid f_i, \rho_0, \rho_1) = (p_i - c)((1-\theta)(a + E(e \mid f_i, \rho_0, \rho_1)) - p_i + \theta E(p_0 \mid f_i, \rho_0, \rho_1))/(1-\theta^2). \]
(A41)

The first-order derivatives of the expected payoffs are as follows:
\[
\frac{\partial E(SS) \mid f_0, \rho_0, \rho_1}{\partial p_0} = (1-\theta)c - p_0 + \theta E(p_1 \mid f_0, \rho_0, \rho_1) \]
\[
\frac{\partial E(\pi_i) \mid f_i, \rho_0, \rho_1}{\partial p_1} = (1-\theta)(a + E(e \mid f_i, \rho_0, \rho_1)) + c + \theta E(p_0 \mid f_i, \rho_0, \rho_1) - 2p_i \]
\[
1-\theta^2 .
\]

Solving the two derivatives as equal to zero, I derive the following expected reaction functions under uncertainty:
\[
\hat{p}_0 = \theta E(\hat{p}_1 \mid f_0, \rho_0, \rho_1) + (1-\theta)c ,
\]
\[
\hat{p}_1 = (\theta E(\hat{p}_0 \mid f_1, \rho_0, \rho_1) + (1-\theta)(a + E(e \mid f_1, \rho_0, \rho_1)) + c)/2 . \]
(A42)

Using Equation (A42), I conjecture that price choices are linear in the information variables, that is, \(\hat{p}_i = X_i^\beta + Y_i^\beta \rho_0 + Z_i^\beta \rho_1 + W_i^\beta f_i\) for each firm \(i = 0, 1.\) Substituting these linear
conjectures of $\hat{p}_0$ and $\hat{p}_1$ into Equation (A42) gives:

$$X_0^B + Y_0^B \rho_0 + Z_0^B \rho_1 + W_0^B f_0 = \theta(X_1^B + Y_1^B \rho_0 + Z_1^B \rho_1 + W_1^B E(f_1 \mid f_0, \rho_0, \rho_1)) + (1 - \theta)c, \quad (A43)$$

$$X_1^B + Y_1^B \rho_0 + Z_1^B \rho_1 + W_1^B f_1 = \left(\theta(X_0^B + Y_0^B \rho_0 + Z_0^B \rho_1 + W_0^B E(f_0 \mid f_1, \rho_0, \rho_1)) + (1 - \theta)(a + E(e \mid f_1, \rho_0, \rho_1)) + c\right)/2. \quad (A44)$$

Combining Equations (A43) and (A44) and using the fact that $E(e \mid f_i, \rho_0, \rho_1) = V(\{\sigma_j + \Delta_j\}f_i + \sigma, \rho_j)/(\sigma_i(\sigma_j + \Delta_j) + V(\sigma_i + \sigma_j + \Delta_j))$ and that $E(f_i \mid f_i, \rho_0, \rho_1) = (V(\Delta_j f_i + (\sigma_i \sigma_j + V(\sigma_i + \sigma_j))\rho_j)/(\sigma_i(\sigma_j + \Delta_j) + V(\sigma_j + \sigma_i + \Delta_j))$ for $(i, j) = (0, 1)$ or $(1, 0)$ yields the following equations.

$$X_0^B = \theta X_0^B + (1 - \theta)c \quad (A45)$$

$$X_1^B = \left((1 - \theta)a + c + \theta X_0^B\right)/2 \quad (A46)$$

$$Y_0^B = \theta Y_1^B \quad (A47)$$

$$Y_1^B = \left((\theta(\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))W_0^B + (1 - \theta)V\sigma_1)/(\sigma_i(\sigma_j + \Delta_j) + V(\sigma_j + \sigma_i + \Delta_j)) + \theta Y_0^B\right)/2 \quad (A48)$$

$$Z_0^B = \left(\theta(\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))W_0^B)/(\sigma_i(\sigma_j + \Delta_j) + V(\sigma_j + \sigma_i + \Delta_j)) + \theta Z_1^B\right)/2 \quad (A49)$$

$$Z_1^B = \theta Z_0^B \quad (A50)$$

$$W_0^B = \theta V_0^B \left(W_1^B / (\sigma_0(\sigma_1 + \Delta_j) + V(\sigma_0 + \sigma_1 + \Delta_j))\right) \quad (A51)$$

$$W_1^B = V\left((1 - \theta)(\sigma_0 + \Delta_j) + V(\sigma_0 + \sigma_1 + \Delta_j))\right)/2(\sigma_j(\sigma_0 + \sigma_i + \Delta_j)) + V(\sigma_0 + \sigma_i + \Delta_j)) \quad (A52)$$

Combining Equations (A45)–(A52) to solve for $(X_0^B, X_1^B, Y_0^B, Y_1^B, Z_0^B, Z_1^B, W_0^B, W_1^B)$ and substituting the solutions into $\hat{p}_i = X_i^B + Y_i^B \rho_0 + Z_i^B \rho_1 + W_i^B f_i \quad (i = 0, 1)$ yields the equilibrium strategies $(\hat{p}_0, \hat{p}_1)$ as shown in this lemma.

Finally, I substitute $(\hat{p}_0, \hat{p}_1), f_i = e + \varepsilon_i$, and $\rho_i = e + \varepsilon_i + \delta_i \quad (i = 0, 1)$ into Equations (A40) and (A41) to yield the unconditional expected payoffs at Date 1. Using $E(e^2) = V, E(\varepsilon_i^2) = \sigma_i, E(\delta_i^2) = \Delta_i$, and $E(e) = E(\varepsilon_i) = E(\delta_i) = 0 \quad (i = 0, 1)$, I derive $E(SS)$ and $E(\pi_i)$ as shown in this lemma. □
Proof of Lemma 5. From Lemma 3, the following inequalities hold.
\[
E(SS^{(D,ND)}) - E(SS^{(ND,ND)}) = (1 - \theta)^2 (3 - \theta^2) \sigma_1^2 V^2 / [2(2 - \theta^2)] (V + \sigma_1) (\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))] > 0
\]
\[
E(\pi_1^{(D,ND)}) - E(\pi_1^{(ND,ND)}) = \theta (8 - 8 \theta + \theta^3) \sigma_1^2 V^2 / [4(2 - \theta^2)] (V + \sigma_0) (\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))] > 0,
\]
proving that this lemma holds. □

Proof of Lemma 6. From Lemma 4, the following inequalities hold:
\[
E(\pi_1^{(D,ND)}) - E(\pi_1^{(ND,ND)}) = \frac{(2 - \theta)(1 - \theta)\theta^2 (2 + \theta)\sigma_0^2}{4(1 + \theta)(2 - \theta^2)} (V + \sigma_0) (V\sigma_0 + V\sigma_1 + \sigma_0 \sigma_1) > 0
\]
\[
E(\pi_1^{(ND,ND)}) - E(\pi_1^{(ND,ND)}) = \frac{(1 - \theta)^2 V^2 (\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))] (2(2 - \theta^2) V^2 + (4 - \theta^2) \sigma_0 \sigma_1 + (4 - \theta^2) V(\sigma_0 + \sigma_1))] (1 + \theta)(2 - \theta^2)] (V + \sigma_0) (2 - \theta^2) V^2 + 2\sigma_0 \sigma_1 + 2V(\sigma_0 + \sigma_1)]^2 > 0.
\]
These two inequalities suggest that the payoff for Firm 1 is higher when the firm discloses information than otherwise, regardless of whether Firm 0 adopts strategy D or ND, indicating that D is the dominant strategy for Firm 1. This suggests that (D, D) and (ND, D) are the candidates for the PBE because D is the dominant strategy for Firm 1. The following inequality indicates that Firm 0 chooses strategy ND if Firm 1 takes strategy D because of the greater expected social welfare.
\[
E(SS^{(ND,ND)}) - E(SS^{(D,ND)}) = (1 - \theta)^2 V^2 \sigma_1^2 / [2(2 - \theta^2)] (V + \sigma_1) (\sigma_0 \sigma_1 + V(\sigma_0 + \sigma_1))] > 0. □
\]

Proof of Proposition 3. Lemma 5 suggests that the candidates for the PBE under Cournot competition are (D, ND) and (ND, ND). If Firm 0 takes strategy ND, Firm 1 chooses a disclosure strategy depending upon the following condition concerning the exogenous parameters.

D if \( \sigma_0 < (1 - \theta)\theta V \sigma_1 / ((2 - \theta)(V + \sigma_1)) \)

ND if \( \sigma_0 > (1 - \theta)\theta V \sigma_1 / ((2 - \theta)(V + \sigma_1)) \)

either D or ND if \( \sigma_0 = (1 - \theta)\theta V \sigma_1 / ((2 - \theta)(V + \sigma_1)) \)

If Firm 1 takes strategy ND, Firm 0 chooses a strategy, depending upon the following condition.

D if \( \sigma_i < \Sigma \) or \( (2 - \theta)\theta V \sigma_0 / (2(1 - \theta)(V + \sigma_0)) < \sigma_i \)

D if \( \sigma_i < \Sigma \) or \( (2 - \theta)\theta V \sigma_0 / (2(1 - \theta)(V + \sigma_0)) < \sigma_i \)
ND if $\Sigma < \sigma_1 < (2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0))$

either D or ND if $\sigma_1 = (2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0))$ or $\sigma_1 = \Sigma$

One may prove that if $\sigma_1 < (2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0))$ is met, then $\sigma_0 > (1 - \theta)\vartheta V \sigma_1 / ((2 - \theta)(V + \sigma_1))$ is automatically satisfied. Therefore, if $\Sigma < \sigma_1 < (2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0))$ holds, then (ND, ND) constitutes the PBE. By contrast, if $\sigma_1 < \Sigma$ or $(2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0)) < \sigma_1$, then (D, ND) constitutes the PBE. Finally, both (D, ND) and (ND, ND) become an equilibrium if $\sigma_1 = (2 - \theta)\vartheta V \sigma_0 / (2(1 - \theta)(V + \sigma_0))$ or $\sigma_1 = \Sigma$. □

**Proof of Proposition 4.** Lemma 6 proves that (ND, D) constitutes the PBE under Bertrand competition. □

**References**


OECD (2011) Corporate Governance of State-Owned Enterprises: Change and Reform in
Table 1. Summary of disclosure strategy in equilibrium under mixed duopoly

<table>
<thead>
<tr>
<th>Uncertain factor</th>
<th>Cost (D, D)</th>
<th>Demand (D, ND) or (ND, ND)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competiton mode</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bertrand</td>
<td></td>
<td></td>
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</tbody>
</table>

Note: The first and second letters in parenthesis represent the disclosure strategies taken by the SOE (Firm 0) and the joint-stock company (Firm 1), respectively.