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A Spatial–Temporal Subspace-based Compressive Channel Estimation Technique in Unknown Interference MIMO Channels

Yasuhiro Takano, Member, IEEE, Hsuan-Jung Su, Senior Member, IEEE, Yoshiaki Shiraishi, Member, IEEE, and Masakatu Morii, Member, IEEE

Abstract—Spatial–temporal (ST) subspace-based channel estimation techniques formulated with $\ell_2$ minimum mean square error (MMSE) criterion alleviate the multi-access interference (MAI) problem when the interested signals exhibit low-rank property. However, the conventional $\ell_2$ ST subspace-based methods suffer from mean squared error (MSE) deterioration in unknown interference channels, due to the difficulty to separate the interested signals from the channel covariance matrices (CCMs) contaminated with unknown interference. As a solution to the problem, we propose a new $\ell_1$ regularized ST channel estimation algorithm by applying the expectation-maximization (EM) algorithm to iteratively examine the signal subspace and the corresponding sparse-supports. The new algorithm updates the CCM independently of the slot-dependent $\ell_1$ regularization, which enables it to correctly perform the sparse-independent component analysis (ICA) with a reasonable complexity order. Simulation results shown in this paper verify that the proposed technique significantly improves MSE performance in unknown interference MIMO channels, and hence, solves the BER floor problems from which the conventional receivers suffer.

Index Terms—Multi-access interference (MAI), unknown interference, subspace-based channel estimation, compressive sensing, principal component analysis (PCA), independent component analysis (ICA).

I. INTRODUCTION

Interference alignment techniques (e.g., [1]) can improve throughput performance in multi-access interference (MAI) channels by exploiting spatial-degrees of freedom (DoF) of multiple-input multiple-output (MIMO) channels if channel state information (CSI) is known accurately. We can utilize $\ell_2$ spatial-temporal (ST) subspace-based channel estimation techniques [2] to obtain accurate enough CSIs. The ST subspace-based techniques alleviate the MAI problem by utilizing a property that the rank of the channel covariance matrix (CCM) [3] of the interested signals is less than the observed dimension. However, in practice, receivers have to estimate channels over unknown interference caused by hidden terminals [4], or artificial noise [5] for secure transmission, etc. The conventional $\ell_2$ techniques can seriously suffer from mean squared error (MSE) deterioration in such scenarios due to the difficulty to separate the interested signals from the CCM contaminated with unknown interference.

Compressed sensing (CS)-based algorithms can be utilized to improve the MAI problem if the interested parameters exhibit sparse nature. This is because they are executed in an observed sub-domain so that the $\ell_0$ or $\ell_1$ norm of the estimate is minimized, which consequently perform interference suppression as well. For example, spatio-temporal compressive channel estimation techniques [6], [7] are proposed for massive MIMO systems. However, they aim to improve multiple-input single-output (MISO) reception performance in downlink and do not assume the spatial DoF of multiple receive antennas.

Recently, MIMO channel estimation algorithms are extensively studied for millimeter-wave (mmWave) systems (e.g., [8]–[11]) by leveraging that channel parameters in narrowband transmission can be approximated to a sparse matrix of the angular domain representation [12]. As shown in [8], an orthogonal matching pursuit (OMP)-based technique outperforms the ordinary $\ell_2$ least squares (LS) technique in a mmWave MIMO system. However, the greedy OMP algorithm does not always achieve its analytical MSE performance since the MSE convergence performance depends significantly on the stopping criterion and the design of the dictionary including beamforming matrices.

A sparse Bayesian learning (SBL)-based algorithm [9] improves estimation accuracy over the OMP by exploiting the block sparsity property [9] commonly observed for the angular domain channel gain vectors in certain $L_M$ measurements. Note that the estimation algorithms [8], [9] do not consider the MAI problem directly. The signal model in both studies is formulated as a collection of received single-input multi-output (SIMO) signals from each transmission (TX) beam. The formulation is reasonable in narrowband transmission, however, it increases pilot overheads in broadband transmission.

For frequency selective fading (FSF) MIMO channels, CS-based channel estimation algorithms are proposed in [10], [11]. Although they consider the estimation problem in MAI channels, the assumption of a long channel coherent time limits application scenarios such as frequency division duplex (FDD)-mmWave systems with a slow mobility and a short transmission time interval (TTI). Moreover, in [10], [11], the MAI problem is not explicitly considered since the uplink TX precoder is supposed to avoid interference between TX beams. However, in a multiuser uplink MIMO scenario, the interference alignment using TX beamforming only cannot perfectly eliminate the MAI [1]. Moreover, not all uplink
Notations

V shows analytical estimation performance of the proposed MIMO transmission system assumed in this paper. Section performance of any Notes that the proposed ℓ1 error rate (BER) floor problem from which the conventional technique improves the MSE performance in the sparse-subspace and the corresponding supports. Simulation results shown in this paper verify that a receiver using the proposed technique improves the MSE performance in unknown interference MIMO channels and, hence, solves a bit error rate (BER) floor problem from which the conventional receivers suffer.

Contributions of this paper are summarized as follows:

- ST subspace-based channel estimation algorithms using the independent component analysis (ICA) [13] are compared with that using the principal component analysis (PCA) [2]. A new ℓ1 regularized ICA estimation algorithm outperforms the conventional ℓ2 PCA approach in unknown interference MIMO channels.
- We show a novel CCM updating algorithm which can be performed independently of the ℓ1 regularization. Note that the sparse-supports, referred to as active-set [14], can be changed over slot-times, which contradicts a requirement that the active-set has to be consistent in a certain duration to correctly perform the sparse-PCA (e.g., [15]) and/or ICA approaches.
- Performance analysis of the ICA-based ST channel estimation is detailed. Based on the analytical performance, we improve the previously-proposed adaptive active-set detection (AAD) technique [16], [17] for the unknown interference MIMO channels.

Note that the proposed ℓ1 ICA approach can improve MSE performance of any ℓ1 LS estimates. This paper shows, however, such a naive extension is not always the optimum.

This paper is organized as follows. Section II clarifies a MIMO transmission system assumed in this paper. Section IV presents the new channel estimation techniques. Section V shows analytical estimation performance of the proposed techniques. Section VI verifies the effectiveness of the new algorithms via computer simulations. Section VII shows concluding remarks.

Notations: The bold lower-case x and upper-case X denote a vector and a matrix, respectively. For matrix X, its transpose and transposed conjugate are denoted as XT and XH, respectively. X−1 and X† denote the matrix inverse and the Moore-Penrose pseudo-inverse of X, respectively. The Cholesky decomposition of X is denoted by XH/2X1/2. [ ] and ( ) are the ceiling and floor functions, respectively. Operators used in this paper are summarized in Table I.

### Table I: Operators

<table>
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<tr>
<th>Operator</th>
<th>Definition</th>
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<tr>
<td>XLA</td>
<td>Submatrix composed of the column vectors in X corresponding to the set A, where the ℓ1-LC font is used for index sets. A set of consecutive numbers {i, \ldots, j} is denoted by A = {i : j}, where {i : j} = \emptyset if i &gt; j.</td>
</tr>
<tr>
<td>JA</td>
<td>Factor matrix JA = [I]A denotes a compressed/sparse matrix, where I is an identity matrix, e.g., X = [x1, x2, x3, x4, x5], A = {1, 3} \Rightarrow XLA \cdot J_A = [x1, x3] \cdot J_A = [x1, 0, x3, 0, 0].</td>
</tr>
<tr>
<td>vec(X)</td>
<td>M \times N × 1 vector composed by stacking the columns of X \in \mathbb{C}^{M \times N}.</td>
</tr>
<tr>
<td>natx(x)</td>
<td>Inversion of the vectorization: natx(vec(X)) = X.</td>
</tr>
<tr>
<td>diag(X)</td>
<td>Vector composed of the diagonal elements of X.</td>
</tr>
<tr>
<td>diag(x)</td>
<td>Diagonal matrix formed with the vector x.</td>
</tr>
<tr>
<td>tplx(r)</td>
<td>M × N Toeplitz matrix whose first row is length N vector r.</td>
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<tr>
<td>|X|_2 |X|_X _B</td>
<td>Weighted Frobenius norm: |X|_F = tr(BXAXH) for X \in \mathbb{C}^{M \times N} with positive definite matrices A and B, Moreover, |X|_X = |X|_X^{1/2} = |X|_2, where J_M is the M × M identity matrix.</td>
</tr>
<tr>
<td>|X|_1</td>
<td>Matrix l1 norm: \sum_{i=1}^{M} \sum_{j=1}^{N}</td>
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<tr>
<td>|X|_2^\Delta _j</td>
<td>Average of matrix sequence X(j) sampled with an interval \Delta = \frac{1}{T_0} \sum_{n=\Delta l \cdots}^{n=\Delta (l+1)} X(j) for the past L observations from the timing l. For L = 1, we denote |X|_2^\Delta _j = |X|_2 _j. Moreover, E[X(j)] = \mathbb{E}[X(j)].</td>
</tr>
<tr>
<td>E[X(j)]</td>
<td>Column-wise covariance matrix: E[X(j)X(j)H] = E[XH].</td>
</tr>
<tr>
<td>\rho(X)</td>
<td>Row-wise covariance matrix: E[X(j)X(j)H] = E[XH].</td>
</tr>
<tr>
<td>H(l)</td>
<td>Projection matrix XXH of X.</td>
</tr>
<tr>
<td>l</td>
<td>The indicator function of X.</td>
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## II. System Model

### A. Received Signal

Consider channel estimation in a TDD-MIMO system composed of NT transmit- and NR receive-antennas. Ls-symbol training sequence (TS) xk, \forall k \in \{1 : NT\}, is transmitted over FSF channels whose channel impulse response (CIR) lengths are at most W symbols. (W−1)-symbol guard interval (GI) is added at the front and rear of TS in order to avoid inter-block interference in the received TS signals of \( L_t = L_s + W - 1 \) symbols. Received TS matrix \( Y(l) = x(l)X + N(l) + Z(l) \), where the timing index \( l \) is a multiple of a slot interval \( L_s \). Channel, TS, additive white Gaussian noise (AWGN) and unknown interference matrices are

\[
H(l) = [H_1(l), \ldots, H_{NT}(l)] \in \mathbb{C}^{N_R \times W \cdot NT},
\]

\[
X = [X_1^T, \ldots, X_{NT}^T]^T \in \mathbb{C}^{W \cdot NT \times L_t},
\]

\[
N(l) = [n_1(l), \ldots, n_{NR}(l)]^T \in \mathbb{C}^{N_R \times L_t},
\]

\[
Z(l) = [z_1(l), \ldots, z_{NR}(l)]^T \in \mathbb{C}^{N_R \times L_t},
\]

respectively. The noise vector at the \( n \)-th receive (Rx) antenna \( n(l) \) follows the Complex normal distribution \( \mathbb{C} \mathbb{N}(0, \sigma_n^2) \) and has the spatially uncorrelated property: \( \mathbb{E}[n(l) n(l)^H] = 0 \) for \( i \neq j \). However, the unknown interference vector \( z_n(l) \) does not always have the spatially uncorrelated property,
since the interference caused by leakage signals from specific hidden terminals is observed via Rx array antennas [18]. Hence, we assume zero-mean and temporally-white properties only: $E[z_n(t)] = 0$ and $\frac{1}{\tau_p} \sum_{n=1}^{N_f} E[z_n(t)z^*_n(t)] = \sigma^2_f I_{L_f}$, respectively. The variances $\sigma^2_f$ and $\sigma^2_p$ per receive antenna are determined according to signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR), respectively.

The TS submatrix $X_k$ is defined by $t p l z^T_W \{ [I_k^T, 0^T_{W-1}] \}$, where the operation $t p l z^T_W \{ r \}$ constructs a $W \times L$ Toeplitz matrix whose first row vector is $r \in \mathbb{C}^{1 \times L}$.

### B. Channel Model

Under the wide-sense stationary uncorrelated scattering (WSSUS) assumption, let the channel coherent time be greater than a slot duration $L_{\text{slot}}T_{\text{sym}}$ [sec], where $L_{\text{slot}}$ and $T_{\text{sym}}$ are the slot length in symbol and the symbol interval in second, respectively. The $(n, j)$-th entry of the CIR matrix $H_k(\tau)$ is composed of $r_k$/resolvable paths, and it is observed via pulse shaping filters $p[t]$, as $h_{k,j}(\tau)[t] = p[t] \ast \left( \sum_{\tau_r=1}^{r_k} b_{k,j}^r(\tau)[t] \delta[\tau - \tau_k^r] \right)$ at time $t = (j + L_{\text{slot}})T_{\text{sym}}$ [sec], where $b_{k,j}^r(\tau)$ and $\tau_k^r$ are the complex gain and path delay of the $r$-th path [19]–[21]. The convolution operator and the Dirac delta function are denoted by $\ast$ and $\delta[t]$, respectively.

As discussed in [2], [16], the $k$-th CCM $H_k(\tau)$ between the $k$-th transmit- and $N_R$ receive-antennas can be written as

$$H_k(\tau) = B_k(\tau)E_k^H,$$

where the matrix $B_k(\tau) \in \mathbb{C}^{N_R \times r_k}$ describes slot-dependent complex gains of $r_k$-resolvable paths. However, the eigenvectors $E_k \in \mathbb{C}^{W \times r_k}$ are independent of slot-time since they can be seen as a time-invariant finite impulse response (FIR) filter representing the response of pulse shaping filters and multipath channels. The gain matrix $B_k(\tau) = [b_{k,i,j}(\tau), \cdots, b_{k,r,j}(\tau)]$ is generated from a scattering cluster-based model, such as the spatial channel model (SCM) [22]. The $i$-th path vector $b_{k,i,j}(\tau)$ can, hence, be written in the angular domain representation [12]:

$$b_{k,i,j}(\tau) = U_{k,i} \cdot a_{k,i,j} \cdot u_{k,i},$$

where the unitary matrices $U_{k,i} \in \mathbb{C}^{N_R \times N_R}$ and $u_{k,i} \in \mathbb{C}^{1 \times 1}$ denote characteristics of propagation channels from a reflector to receive- and, to transmit-antennas, respectively. The vector $a_{k,i,j}(\tau)$ follows a Rayleigh distribution. Note that the receiver has no prior knowledge of the hyper parameters $\{r_k, E_k, U_{k,i} \}$, except that the expected trace of $H_k(l)$ is $E[\|H_k(l)\|^2] = \sigma^2_H$ with a constant $\sigma^2_H$.

### III. PRELIMINARIES

#### A. Channel Covariance Matrices

Let us compute CCMs of (2) and (3). By performing the singular value decomposition (SVD), we have

$$V_k D_k V_k^H = K[H_k(j)],$$

$$U_{P,k} D_{P,k} U_{P,k}^H = R[H_k(j)],$$

$$U_{k,i} D_{k,i} U_{k,i}^H = R[b_{k,i,j}(j)],$$

where $V_k \in \mathbb{C}^{W \times W}$, $U_{P,k} \in \mathbb{C}^{N_R \times N_R}$ and $U_{k,i} \in \mathbb{C}^{N_R \times N_R}$ are unitary matrices. We denote column-wise and row-wise covariance matrices for a sequence $X(j)$ by $K[\mathbf{X}(j)] = E[\mathbf{X}(j)\mathbf{X}^H(j)]$ and $R[\mathbf{X}(j)] = E[\mathbf{X}(j)\mathbf{X}^H(j)]$, respectively. Diagonal matrices $D_{T,k}$, $D_{P,k}$ and $D_{k,i}$ are constructed from singular values of the corresponding covariance matrices, respectively. We refer to (4) as temporal CCM. The spatial versions (5) and (6) are referred to as principal (p-)spatial, and independent (i-)spatial CCMs, respectively, by following concept of the ICA [13]. Moreover, joint subspaces reduced from (4), (5) and (6), respectively, referred to as pST and iST subspaces, hereafter.

#### B. Channel Rank Properties

The reduced-rank channel estimation techniques (e.g., [2]) utilizes a property that the rank of the interested signals in the noisy CCMs is less than the original dimension. We define adaptive-ranks for the CCMs by generalizing that shown in [17], where the following notations are used. For an $N \times M$ parameter matrix $A$, let $\tilde{A}$ denote a noisy observation: $A = A + N_A$, where the matrix $N_A$ has properties: $\frac{1}{M}E[N_A] = \sigma^2_N I_N$ and $\frac{1}{N}E[N_A] = \sigma^2_N I_M$ with a constant $\sigma^2_N$.

**Definition 1** (Adaptive-ranks). In the temporal CCM of $\tilde{A}$, the dimension of the interested signals above the noise level $\sigma^2_N$ can be approximated in a subspace of the dimension given by the temporal adaptive-rank:

$$\text{ar}(\tilde{A}, \sigma^2_N) = \sum_{\gamma_n} \mathcal{I}(\lambda_{\sigma^2_N}(\tilde{A}) \geq \gamma N^2),$$

where $\mathcal{I}\{\cdot\}$ is the indicator function and $\lambda_{\sigma^2_N}(\mathbf{M})$ denotes the $n$-th singular value of matrix $\mathbf{M}$. Moreover, $\gamma \overset{\text{def}}{=} \min \left\{ \text{ar}(\tilde{A}, \sigma^2_N), \text{ar}(\mathbb{R}[\tilde{A}], \sigma^2_N) \right\}$, where the spatial adaptive-rank is given by

$$\text{ar}(\mathbb{R}[\tilde{A}], \sigma^2_N) = \text{ar}(\mathbb{R}[\tilde{A}], \sigma^2_N).$$

The parameter $\gamma$ is, similar to [12], (7.44), the rank of the signal matrix in $A$ corresponding to the joint spatial-temporal subspace. The adaptive-ranks are determined by iteratively examining (7) and (8), where we initialize $\gamma = \min(M, N)$.

Let us see adaptive-ranks for CCMs of CIR estimates. Suppose that the receiver estimates the CIR as $H_k(j) = H_k(j) + N_{hk}$ using a length $L_t$ TS, after relevant noise whitening transformations for the received signals such that $\text{vec}(N_{hk}) \sim \mathcal{CN}(0, \frac{\sigma^2_N}{L_t} I_{W^2} \otimes I_{N_R})$. The operator $\otimes$ is Kroenecker product. The variance $\sigma^2_N \overset{\text{def}}{=} \sigma^2_f + \sigma^2_p$ is given according to the received signal-to-interference-plus-noise ratio (SINR). We define adaptive-ranks corresponding to (4), (5) and (6), as $r_{T,k} = \text{ar}(K[H_k(j)], \sigma^2_N)$, $r_{P,k} = \text{ar}(R[H_k(j)], \sigma^2_N)$, and $r_{k,i} = \text{ar}(R[b_{k,i,j}(j)], \sigma^2_N)$, respectively.

**Property 1** (Bounds of the adaptive-ranks),

$$r_{T,k} \leq W,$$

$$r_{P,k} \leq N_R,$$

$$r_{t,k,i} \leq r_{p,k}.$$
Proof. We prove (11) only, since (9) and (10) are obvious. Let us omit the indexes \( k \) of TX streams and \( j \) of slot timings for the sake of simplicity. Moreover, we may assume \( r_{T,P} \geq r_{T} \) in spatially dense large-scale MIMO channels. By [23, Corollary 3.4.3], we have \( \sum_{n=1}^{N} \sum_{i=1}^{L_{n}} \lambda_{n}^{2}(R[R_{i}]) \geq \sum_{n=1}^{N} \lambda_{n}^{2}(\sum_{i=1}^{L_{n}} R[R_{i}]) \geq N \sum_{n=1}^{N} \lambda_{n}^{2}(\sum_{i=1}^{L_{n}} R[R_{i}]) \geq N \sum_{n=1}^{N} \lambda_{n}^{2}(R[R_{i}]) \) for \( N \leq N_{R} \). Taking average for \( r_{T} \) paths yields \( E \left[ \sum_{n=1}^{N} \sum_{i=1}^{L_{n}} \lambda_{n}^{2}(R[R_{i}]) \right] \geq \frac{1}{r_{T}} \sum_{n=1}^{N} \sum_{i=1}^{L_{n}} \lambda_{n}^{2}(R[R_{i}]) \), since \( R[H] = R[B] = \sum_{n=1}^{N} \sum_{i=1}^{L_{n}} R[R_{i}] \) according to (2) and (3). Because the equality always holds for \( N = N_{R} \), we find that the variance of vector \( \{ \lambda_{n}^{2}(R[R_{i}]) \} \) is not less than that of \( \{ \lambda_{n}^{2}(R[R_{i}]) \} \). Hence, for the threshold \( \sigma^{2}_{H}/L_{t} \), \( \bar{r}_{l,i} \) is \( \frac{1}{r_{T}} \sum_{n=1}^{N} \sum_{i=1}^{L_{n}} \frac{\lambda_{n}^{2}(R[R_{i}])}{\lambda_{n}^{2}(R[R_{i}])} \geq \frac{\sigma^{2}_{H}}{r_{T}^{2}} \). \( \square \)

Proposition 1 (Ranks of joint ST subspaces). The dimension of the iST subspace is less than that of the pST subspace.

Proof. \( \sum_{i=1}^{r_{T,P}} r_{l,k,i} = r_{T,k} r_{P,k} \leq r_{T,k} r_{P,k} \) for \( \forall i \). \( \square \)

C. Examples

Fig. 1 illustrates Proposition 1, where CIR matrices \( H_{k} \) follow the Pedestrian-B (PB) model [22] assuming SIMO channels with \( N_{R} = 12 \) antennas. As observed from Fig. 1(a), the delay profile \( \delta \{ \epsilon_{i} [R[H_{i}]] \} \) spreads over \( W = 31 \) symbols. However, we confirm from Fig. 1(b) that the temporal adaptive-rank is \( r_{T,k} = 5 \leq W \) for a noise level \( \sigma_{n}^{2} = 10^{-3} \).

Fig. 1(c) shows spatial profile vectors \( \delta \{ \epsilon_{i} [R[H_{i}]] \} \) and \( \delta \{ \epsilon_{i} [R[B_{i}]] \} \). Unlike the delay profile, the spatial profiles exhibit dense nature in the observed domain. Indeed, as shown in Fig. 1(d), the p-spatial singular values \( \lambda_{i}^{2}[R[H]] \) are supported in almost \( N_{R} \) dimensions, which means that, in the SCM-based channels, the conventional spatial-spread reduction techniques (e.g., [2]) cannot always obtain significant improvement in a high SINR regime.

Nevertheless, we aim to estimate CIRs in unknown interference channels. As observed from Fig. 1(d), the number of significant singular values \( \lambda_{i}^{2}[R[H]] \) is \( r_{T,k} = 6 \) for the noise level \( \sigma_{n}^{2} = 10^{-3} \). However, in the iST subspace, the singular values \( \lambda_{i}^{2}[R[B_{i}]] \) are counted as \( \sum_{i=1}^{r_{T,P}} r_{l,k,i} = 22 \leq r_{T,k} r_{P,k} = 30 \).

IV. CHANNEL ESTIMATION

A. Problem Formulation

An \( \ell \) regularized ST-MMSE channel estimation problem for TDD reception with slot-interval \( \Delta_{T} \) is written as

\[
\hat{H}^{\ell}_{ST}(l) = \arg \min_{H(l)} \mathbb{E}_{L_{T}} \left[ L(j, H(j)) + \| \xi \|_{1} \right]
\]  \quad (12)

with a Lagrange multiplier \( \xi \) [24], where \( \| \cdot \|_{1} \) is the matrix \( \ell \) norm. We denote the expectation operation as \( \mathbb{E}_{L_{T}} [ s(j) \| s(j) \| ] = \sum_{s(j) = \Lambda_{T}} s(j) \right \}$ for a length-\( L \) sequence \( s(j) \) sampled with an interval \( \Lambda \). The log-likelihood function \( \ell(j) \) is

\[
\ell(j, H) = \frac{1}{\sigma^{2}_{\Gamma}} \| Y(j) - HX(j) \|_{2}^{2} + \Gamma.
\]  \quad (13)

The spatial weight matrix \( \Gamma \) is defined as \( \Gamma = \sigma^{2}_{\Gamma} R_{\Gamma^{-1,\Gamma}} \), where \( R_{\Gamma} = \frac{1}{L_{T}} \mathbb{E}_{L_{T}} [ \Psi(j)] \) with \( \Psi(j) = N(j) + Z(j) \). We may use \( R_{\Gamma} = (1 - \alpha) \sigma^{2}_{\Gamma} N_{R} + \alpha \frac{1}{L_{T}} \mathbb{E}_{L_{T}} [ \Delta_{T}^{Y(j) - H(j)} X(j)] \), \quad (14)

where the parameter \( \alpha \) is given by \( \sigma^{2}_{\Gamma} \Delta_{T}^{Y(j) - H(j)} X(j) \), so that the SINR of \( R_{\Gamma} \) is consistent to \( \sigma^{2}_{\Gamma} \).

The problem (12) aims to find a temporally sparse solution \( \hat{H}_{A}(l) \) supported with column indexes \( A \) referred to as active-set. The problem (12) can, hence, be reformulated as an EM problem composed of the following E- and M-steps:

\[
\begin{align*}
\mathbb{G}_{A[n]}(l) & = \arg \min_{G_{A}[n]} \mathbb{E}_{L_{T}} \left[ L(j, G_{A}[n](j)JT_{[n]}(j) | A[n]) \right] \quad (15) \\
A[n+1] & = \arg \min_{A[n+1]} \mathbb{E}_{L_{T}} \left[ \| G_{A}[n+1](j)JT_{A[n+1]}(j) - H(j) \|^{2} \right] \quad (16)
\end{align*}
\]

with \( J_{A} = I_{W_{N_{R}}, A} \), where \( G_{A}(j) = H(j)J_{A} \in \mathbb{C}^{N_{R} \times |A|} \) is a column-shrunken CIR matrix supported with active-set \( A \).

As depicted in Fig. 2, we iteratively perform the pair of sub-problems for predefined constant \( N_{\text{AAd}} \) times, and determine the optimal solution by using corrected-Akaike information criterion (AICc) [25], where the active-set is initialized at \( A[0] = \{ 1 : W_{N_{R}} \} \).

B. Solution to the Conditional MMSE Problem

For the sake of simplicity, let us omit the slot interval \( \Delta_{T} \). The active-set \( A[n] \) at the \( n \)-th EM iteration is abbreviated as \( A \) in Sections IV-B and IV-C. The problem (15) is rewritten [26] as, given active-set \( A \),

\[
\begin{align*}
\mathbb{G}_{A}(l) & = \arg \min_{G_{A}} \mathbb{E}_{L_{T}} \left[ \| G_{A}(j) - G_{A}(j)JT_{[n]}(j) \|^{2} R_{XX}[X_{A}] \right] \quad (17) \\
G_{A}(j) & = \arg \min_{G_{A}} \mathbb{E}_{L_{T}} \left[ \| G_{A}(j) - G_{A}(j)JT_{[n]}(j) \|^{2} R_{XX}[X_{A}] \right] \quad (18)
\end{align*}
\]

where \( R_{XX}(j) = Y(j)X^{H} \) and \( R_{XX}[X_{A}] = J_{A}^{H}XX^{H}J_{A} \). According to [27], we can solve the MMSE problem (17) by using the PCA. In order to perform the PCA accurately, first of all, we transform the problem, as

\[
\begin{align*}
\hat{G}_{A}(j) & = \arg \min_{G_{A}} \mathbb{E}_{L_{T}} \left[ \| \hat{G}_{A}(j) - \hat{G}_{A}(j)JT_{[n]}(j) \|^{2} R_{XX}[X_{A}] \right] \quad (19) \\
\hat{G}_{A}(j) & = \arg \min_{G_{A}} \mathbb{E}_{L_{T}} \left[ \| \hat{G}_{A}(j) - \hat{G}_{A}(j)JT_{[n]}(j) \|^{2} R_{XX}[X_{A}] \right] \quad (20)
\end{align*}
\]

where the noise whitening operation \( \mathbb{W} \{ \cdot \} \) is defined by

\[
\mathbb{W} \{ M \} = \Gamma^{1/2} \left( MR_{XX}[X_{A}] - G_{A}(j) \nabla R_{XX}[X_{A}] \right) \quad (21)
\]
The notation $\mathbf{M}_I$ represents a submatrix of $\mathbf{M}$ specified by the column index set $I$. The temporal subspace $\mathbf{E}_k$ in (2) corresponds to $\tilde{\mathbf{V}}_{A_k} |_{1:M_{\text{ST}}}$ in the transformed space by (21), where $r_{T,k}^\text{MDL}$ is estimated by the minimum description length (MDL) [28] of the vector $\text{diag}(\tilde{\mathbf{D}}^\text{TM}_{A_k})$.

2) iST solution: In (2), the slot-independent gain is written as $\mathbf{B}_k(l) = \mathbf{H}_k(l) \mathbf{E}_k$. Similarly, we define a coarse-estimate gain matrix by

$$\hat{\mathbf{B}}_{T,k}(l) = \hat{\mathbf{G}}_{A_k}^\text{LS}(l) (\tilde{\mathbf{V}}_{A_k} |_{1:M_{\text{ST}}})$$

(25)

for a pre-defined constant $r_{T}^\text{max}$. We obtain a fine-estimate gain $\tilde{\mathbf{B}}_k(l)$ by applying i-spatial subspace projections to each column vector in $\hat{\mathbf{B}}_{T,k}$ that corresponds to (3):

$$\text{vec}(\hat{\mathbf{B}}_{k}(l)) = \tilde{\Phi}_k \text{vec}(\hat{\mathbf{B}}_{T,k}(l)).$$

(26)

The $N_R r_{T}^\text{max} \times N_R r_{T}^\text{max}$ projection matrix $\tilde{\Phi}_k$ is given by

$$\tilde{\Phi}_k = G_{r_{T}^\text{max}}^{L_{\text{max}}} \tilde{\Phi}_{r_{T}^\text{max}}$$

(27)

with $\tilde{\Phi}_{r_{T}^\text{max}} = \mathbf{P}(\hat{\mathbf{U}}_{k,3} |_{1:r_{T}^\text{max}})$, where the unitary matrix $\hat{\mathbf{U}}_{k,3}$ is obtained from the SVD:

$$\hat{\mathbf{U}}_{k,3} \hat{\mathbf{D}}_{k,3} \hat{\mathbf{V}}_{k,3}^\text{H} = R_{k,3} L_{k,3} \left[ \hat{\mathbf{b}}_{k,3}^T(j) \right]$$

(28)

with $\hat{\mathbf{b}}_{k,3}(j) = \hat{\mathbf{B}}_{T,k}(j) |_{j}$. The parameter $r_{T,k,3}$ can be determined by using the MDL of the singular values $\text{diag}(\hat{\mathbf{D}}_{k,3})$.

The estimate (17) for the $k$-th TX stream is written as $\Gamma^{-1/2} \mathbf{G}_{A_k}(l) \mathbf{Q}_{A_k}^\text{M} |_{1:k}$ by performing inversion of (21), where $\hat{\mathbf{G}}_{A_k}(l)$ is obtained by using (26) and (23). Specifically, the iST solution is given by

$$\mathbf{G}_{A_k}(l) = \Gamma^{-1/2} (\mathbf{B}_k(l) |_{1:r_{T,k}}) (\hat{\mathbf{V}}_{A_k} |_{1:r_{T,k}})^\text{H} \mathbf{Q}_{A_k}^\text{H} |_{1:k}.$$  

(29)

where $r_{T,k} \overset{\text{def}}{=} r_{\text{MDL}}^\text{L_{max}} + r_{\Delta}$ with a parameter $3 r_{\Delta}$.

3) pST solution: The pST subspace-based channel estimate [2, 27] can be written in the same form as (29). Specifically, we replace $\mathbf{B}_k(l)$ with $\tilde{\Phi}_{P,A_k} \tilde{\mathbf{G}}_{A_k}^\text{LS}(l) \tilde{\mathbf{V}}_{A_k}$, where the p-spatial projection $\tilde{\Phi}_{P,A_k}$ is obtained from the PCA of the covariance matrix $\text{diag}(\mathbf{G}_{A_k}^\text{LS}(j))$.

**Remark:** In the right-hand side (RHS) of (23), the active-subset $A_k$ has to be consistent over the past $L_{\text{S}}$ slot durations.

The constant is chosen so that $r_{T}^\text{max} \gg r_{P,k}$ for all $k$, since the rank estimate $r_{T}^\text{MDL}$ can change over slot-timings. Hence, we need to update all possible i-spatial CCMs corresponding to the first $r_{T}^\text{max}$-largest paths.  

$r_{\Delta}$ is used for the M-step (16). However, $r_{\Delta} = 0$ in the final E-step.
That is, calculating the temporal covariance matrix (23) directly from (20) incurs two problems:

\begin{itemize}
\item[P1] non-polynomial (NP)-hard, and
\item[P2] the operation (21) requires the CIR to be estimated.
\end{itemize}

Note that the active-subset \( A_k \) has to be known \textit{a priori} although it can be changed in the middle of the transmission. We may take a greedy approach that computes the CCM (23) for all possible \( A_k \). However, it causes Problem P1 since the number of all possible active-sets is of binomial order.

\subsection*{C. New Solutions to the E-step (15)}

This subsection shows, first of all, a solution to Problems P1 and P2 which enables to specify the active-set \textit{a posteriori}. We propose, then, the solution to the E-step (15), referred to as \( \ell_1 \) pST technique, by performing the PCA for p-spatial and temporal subspaces. After that, another new conditional \( \ell_1 \) sST channel estimation using the ICA approach is presented by showing modifications from the \( \ell_1 \) pST algorithm.

1) Temporal CCM: We consider Proposition 2 that computes a temporal covariance for all TX streams. For the sake of simplicity, the slot timings \( l \) and \( m \) are omitted.

\textbf{Proposition 2} (Temporal covariance matrix of (20)).

\[
\begin{align*}
\mathbb{K}\left[ \mathbf{\hat{G}}_{LS}^{T} \right] &= C_{K_A}(A) + C_{K_B}(A) - \{ C_{K_C}(A) + C_{K_C^T}(A) \},
\end{align*}
\]

where

\[
\begin{align*}
C_{K_A}(A) &= \mathbf{R}_{XX;A}^{H/2} \mathbf{J}_A [\mathbf{\hat{R}}_{YY;A}] \mathbf{J}_A \mathbf{R}_{XX;A}, \\
C_{K_B}(A) &= \nabla \mathbf{R}_{XX;A}^{1/2} \mathbf{J}_A [\mathbf{H}] \mathbf{J}_A \nabla \mathbf{R}_{XX;A}^{H/2}, \\
C_{K_C}(A) &= \mathbf{R}_{XX;A}^{H/2} \mathbf{J}_A \mathbf{E}[\mathbf{\hat{R}}_{YY;A} \mathbf{H}] \mathbf{J}_A \mathbf{R}_{XX}^{H/2},
\end{align*}
\]

with \( \mathbf{\hat{R}}_{YY} = \mathbf{\Gamma}^{1/2} \mathbf{R}_{YY} \) and \( \mathbf{H} = \mathbf{\Gamma}^{1/2} \mathbf{H} \).

\textbf{Proof.} Substituting (20) into \( \mathbb{K}\left[ \mathbf{\hat{G}}_{LS}^{T} \right] = \mathbb{E}\left[ \{ \mathbf{\hat{G}}_{LS}^{T} \}^{H} \mathbf{\hat{G}}_{LS} \right] \) obtains Proposition 2.

Proposition 2 shows that the temporal CCM of the compressed estimate (20) can be written as a function of active-set \( A \). However, all the auto/cross-correlation \( \mathbb{K} \) terms are independent of \( A \). We can, thereby, perform sparse-PCA given active-set \( A \), after updating the correlation matrices. Hence, Proposition 2 solves Problem P1 since it is not necessary to compute \( \mathbb{K}\left[ \mathbf{\hat{G}}_{LS}^{T} \right] \) for all possible active-sets \( \forall A \).

Nevertheless, Proposition 2 still has Problem P2. We show the next proposition to be used in a solution of the P2.

\textbf{Proposition 3.} The temporal covariance matrix of the k-th TX stream can be computed as

\[
\begin{align*}
\mathbb{K}\left[ \mathbf{\hat{G}}_{LS}^{T} \right] &= C_{K_A}(A, k) + C_{K_B}(A, k) \\
&- \{ C_{K_C}(A, k) + C_{K_C^T}(A, k) \},
\end{align*}
\]

where

\[
\begin{align*}
C_{K_A}(A, k) &= \sum_{j=1}^{k} \sum_{i=1}^{k} \Omega_{A_i,k}^{H} J_{A_i,k}^T \mathbf{K}_{i,j}^a J_{A_k} \Omega_{A_j,k}^{1}, \\
C_{K_B}(A, k) &= \sum_{j=1}^{k} \sum_{i=1}^{k} \Omega_{A_i,k}^{H} J_{A_i,k}^T \mathbf{K}_{i,j}^b J_{A_k} \Omega_{A_j,k}^{H}, \\
C_{K_C}(A, k) &= \sum_{j=1}^{k} \sum_{i=1}^{k} \Omega_{A_i,k}^{H} J_{A_i,k}^T \mathbf{K}_{i,j}^c J_{A_k} \Omega_{A_j,k}^{H},
\end{align*}
\]

The submatrices \( \mathbf{K}_{i,j}^a, \mathbf{K}_{i,j}^b, \) and \( \mathbf{K}_{i,j}^c \) can be updated independently of the active-set \( A \):

\[
\begin{align*}
\mathbf{K}_{i,j}^a &= \mathbb{E}[\mathbf{\hat{R}}_{YY;A} \mathbf{R}_{YY;A}], \\
\mathbf{K}_{i,j}^b &= \mathbb{E}[\mathbf{H}_i^{H} \hat{\mathbf{H}}], \\
\mathbf{K}_{i,j}^c &= \mathbb{E}[\mathbf{\hat{R}}_{YY;A} \mathbf{H}_i],
\end{align*}
\]

where \( \mathbf{R}_{YY;A} = \mathbf{R}_{YY} | A \). The \( |A_i| \times |A_j| \) matrix \( \Omega_{A_i,j}^{-1} \) is the \( (i, j) \)-th block submatrix of \( \mathbf{R}_{XX}^{1/2} \). Multiplying the \( |W| \times |A_k| \) matrix \( \mathbf{J}_{A_k} = \mathbf{J}_{A_k}^T \mathbf{J}_{A_k} \) extracts the entries corresponding to \( A_k \) from the domain \( \mathbb{D}_k \) = \{ 1 : W \} + (k - 1)W \).

\textbf{Proof.} We can obtain (30) from Proposition 2, by focusing on the \( (k, k) \)-th block matrix.

We find from (32) and (33) that the CIR submatrices \( \mathbf{H}_k \) are required only for \( i \geq k + 1 \) in order to calculate the k-th covariance matrix \( \mathbb{K}\left[ \mathbf{G}_{LS}^{T} \right] \). Therefore, Problem P2 can be solved by the back-substitution algorithm (27) which estimates the TX stream-wise CIRs by descending order of \( k = N_T, \ldots, \) 1 using approximations \( \mathbf{H}_k \approx \mathbf{H}_A | \mathbb{D}_k : \forall k' > k \). Specifically, for the k-th TX stream, we can partially update covariance submatrices \( \mathbf{K}_{i,j}^b, \) and \( \mathbf{K}_{i,j}^c \), as

\[
\begin{align*}
\mathbf{K}_{i,j}^{b, k+1} &= \mathbb{E}[\mathbf{H}_i^{H} \mathbf{H}_j], \\
\mathbf{K}_{i,j}^{c, k+1} &= \mathbb{E}[\mathbf{\hat{R}}_{YY;A} \mathbf{H}_i],
\end{align*}
\]

respectively, by removing terms already calculated for the \( k' \geq (k+1) \)-th covariance matrices \( \mathbb{C}_{K_A}(A, k') \) and \( \mathbb{C}_{K_C}(A, k') \).

2) P-spatial CCM : Proposition 3 can be extended for the p-spatial covariance matrices (5) of the compressed version using the vectorization operation.

\textbf{Proposition 4.} The p-spatial covariance matrix of the k-th TX stream can be computed as

\[
\begin{align*}
\mathbb{R}\left[ \mathbf{\hat{G}}_{AK} \right] &= C_{RA}(A, k) + C_{RB}(A, k) \\
&- \{ C_{RC}(A, k) + C_{RC}^T(A, k) \}.
\end{align*}
\]

We obtain the three matrices in the left-hand side (LHS) of (39) via their vectorized versions, as

\[
\begin{align*}
\text{vec}\{ C_{RA}(A, k) \} &= \mathbf{T}_a \mathbf{v}_A(A, k), \\
\text{vec}\{ C_{RB}(A, k) \} &= \mathbf{T}_b \mathbf{v}_B(A, k), \\
\text{vec}\{ C_{RC}(A, k) \} &= \mathbf{T}_c \mathbf{v}_C(A, k),
\end{align*}
\]

\textbf{Proof.}
where we denote $T_a = E[\hat{R}_{XY} \otimes \hat{R}_{XY}]$, $T_b = E[H^* \otimes \hat{H}]$, $T_c = E[H^* \otimes R_{XY}]$, and

\[
\nu_a(A, k) = \text{vec}(J_A R_{XX}^{1/2} E(A, k) R_{XX}^{1/2} J_A^T),
\]

\[
\nu_b(A, k) = \text{vec}(J_A V R_{XX}^{1/2} E(A, k) \nu V R_{XX}^{1/2} J_A^T),
\]

\[
\nu_c(A, k) = \text{vec}(J_A R_{XX}^{1/2} E(A, k) \nu^2 R_{XX}^{1/2} J_A^T)
\]

with $E(A, k) = D_{2A}((\|i\| \in A_k) \cap \{\|i\| \in A\})^T \in Z^{[A] \times [A]}$.

The back-substitution can be applied to Proposition 4, too.

3) Conditional $\ell 1$ pST algorithm: By combining Sections IV-C1 and IV-C2, the conditional $\ell 1$ pST channel estimation given active-set $A$: $f_{11}^{pST}(Y, R_{XY}, \theta_T, \theta_K \mid A)$ is summarized in Algorithm 1. The input sets $\theta_T$ and $\theta_K$ are

\[
\theta_T = \{T_a, T_b, T_c\},
\]

\[
\theta_K = \{K_{i,j}^a, K_{i,j}^b, K_{i,j}^c \mid 1 \leq i, j \leq N_T\}.
\]

For the sake of conciseness, the known TS matrix $X$ is omitted from input parameters. The parameters $T_a$ and $K_{i,j}^a$ can be updated before executing Algorithm 1, whereas the others are updated at Steps 4 and 7.

The $\ell 1$ MMSE channel estimate (29) is computed at Step 10, by the descending order of $k = N_T, \cdots, 1$ to perform the back-substitution, where the temporal subspace estimate $\hat{V}_{Ak}$ and the spatial projection $\hat{A}_{Ak}$ are obtained at Steps 6 and 9, respectively. It should be emphasized that, at Steps 5 and 8, the temporal and spatial CCMs are reduced via Propositions 3 and 4, but they do not directly require the compressed LS estimate $\hat{G}_{Ak}^LS(l)$.

The output CIR estimate matrix $\hat{H}_{Ak}$ is written as

\[
\hat{H}_{Ak} = [G_{Ak}, \cdots, G_{Ak_N}],]

by using (29). Set $\theta_T$ is output as

\[
\theta_T = \{\hat{V}_{Ak}, \hat{R}_{S_k}, \hat{R}_{T,k} \mid k = 1, \cdots, N_T\}.
\]

Obtain the projector $\hat{P}_{Ak}$ corresponding to (5).

Obtain the $k$-th channel estimate $\hat{G}_{Ak}$ by (29), where $\hat{B}_{Ak} = \hat{P}_{Ak} \cdot \hat{G}_{Ak}^LS \cdot \hat{V}_{Ak}$.

11) end for

Output: $\hat{H}_{Ak}$, $\theta_T$, $\theta_K$ and $\theta_K$.

- Algorithm 2 $f_{11}^{pST}(Y, R_{XY}, \theta_R, \theta_K \mid A)$.

1. Compute the conditional $\ell 1$ LS estimate $\hat{G}_{Ak}^LS$ (18).

2. for $k = N_T$ to 1 do

3. Compute $\hat{G}_{Ak}^LS$ by (24), where $G_{Ak} = \hat{G}_{Ak}$ for $\forall i < k$.

4. Update $\theta_K$ partially by (37) and (38).

5. Update $\theta_T$ by using (30).

6. Obtain the temporal subspace $\hat{V}_{Ak}$ by (23).

7. Update $\theta_T$ partially using (37) and (38).

8. Update $\hat{G}_{Ak}$ by using (39).

9. Obtain the projector $\hat{P}_{Ak}$ corresponding to (5).

10. Obtain the $k$-th channel estimate $\hat{G}_{Ak}$ by (29), where $\hat{B}_{Ak} = \hat{P}_{Ak} \cdot \hat{G}_{Ak}^LS \cdot \hat{V}_{Ak}$.

11) end for

Output: $\hat{H}_{Ak}$, $\theta_T$, $\theta_R$, and $\theta_K$.

D. Solution to the $M$-step (16)

We solve the problem (16) by extending the AAD algorithm [17] to optimize MSE performance of the conditional $\ell 1$ ST channel estimation. In the following, let us denote $\hat{R}_{S,k,i} = \hat{R}_{I,k,i}$, since the AAD algorithm for the $\ell 1$ pST approach is also obtained by assuming $\hat{R}_{S,k,i} = \hat{R}_{I,k,i}$ for $\forall i$. The AAD updates the active-set recursively by

\[
A^{[n+1]} = A^{[n]} \cap \text{AAD}(\hat{H}_{1n}^2, \hat{H}_{1n}^1, \sigma_{[n]}^1 \mid A^{[n]})
\]

\[
\mathcal{J}_{[n]} \mid \mathcal{A}^{[n]} > 0, \forall j \in \mathcal{A}^{[n]} \cap T_{W}(\Delta \hat{d}^{[n]}, \mathcal{E})
\]

where the superscript $[n]$ denotes $n$-th iteration. The parameter $\Delta \hat{d}^{[n]}$ is the $j$-th entry of residue vector $\Delta \hat{d}^{[n]} = \hat{d}^{[n]} - \{m^{[n]}(\sigma_{[n]}^1) + e^{[n]}(\sigma_{[n]}^2)\}$, where the delay profile estimate is

\[
\hat{d}^{[n]} = \begin{cases} \text{diag}(\hat{R}_{S,k}^T \cdot \hat{H}_{2n}(j)) \quad (n = 0) \\ \text{diag}(\hat{H}_{1n}^2(\hat{H}_{1n}^1)) \quad (n \geq 1) \end{cases}
\]
The vector \( m^{[n]}(\sigma^{2}_n) \) approximately describes the distribution of the squared errors over CIR taps for the channel estimate \( \hat{H}^{[n]}_{\ell} : [m^{[n]}_1(\sigma^{2}_n)^T, \ldots, m^{[n]}_K(\sigma^{2}_n)^T]^T \), where \( m^{[n]}_k(\sigma^{2}_n) \) \( \def \{ J^{[n]}_k \} \) \( diag \{ \hat{K}^{[n]}_{Gk} (\sigma^{2}_n, A^{[n]}_k) \} \) with

\[
K^{[n]}_{Gk} (\sigma^{2}_n, A^{[n]}_k) = \sigma^{2}_n \hat{Q}^{-1} \hat{V}_{Ak} \hat{A}_{S,k} \hat{S}_{k} \hat{V}_{Ak} Q^{-1} \hat{A}_{k}.
\]

We denote \( \hat{V}_{Ak} = \hat{V}_{Ak} \sideset{\|}{1}_{Ak} \) and \( \hat{A}_{S,k} = \diag(\hat{r}_{Ak}) \). The error \( e^{[n]}(\sigma^{2}_n) \) vector of the estimated delay profile \( \hat{d}^{[n]}_H \) may be approximated by \( \{ e^{[n]}_1(\sigma^{2}_n)^T, \ldots, e^{[n]}_{N_T}(\sigma^{2}_n)^T \}^T \), where \( e^{[n]}_k(\sigma^{2}_n) = \mu(\sigma^{2}_n, R_{S,k}, \hat{r}_{T,k}, A^{[n]}_k) \diag \{ J^{[n]}_k \} / |A^{[n]}_k|. \)

\[
(45)
\]

The function \( \mu(\cdot) \) represents, as discussed later in Section V, an MSE performance of the compressed estimate \( G^{[n]}_{\ell} \):

\[
\mu(\sigma^{2}_n, R_{S,k}, \hat{r}_{T,k}, A^{[n]}_k) = tr \{ K^{[n]}_{Gk} (\sigma^{2}_n, A^{[n]}_k) \} = \sigma^{2}_n \sum_{k=1}^{K} R_{S,k} tr \{ R^{-1}_{XX,k} \} / |A^{[n]}_k|. \)

\[
(46)
\]

The operation \( T_W(\mathbf{x}, E) \) forms an index subset of the top \( E \) entries in each length-\( W \)-subvector of vector \( \mathbf{x} \) for a predefined constant \( E \leq W \), which imposes the maximum cardinality regularization onto the active-set.

**E. Solutions to (12)**

We show a new (unconditional) \( \ell_1 \) iST channel estimation algorithm. Notice that the (unconditional) \( \ell_1 \) pST algorithm is straightforwardly obtained by replacing the conditional ICA \( f^{\ell_1}_{ICA}(\cdot | A) \) with the PCA version \( f^{\ell_1}_{PCA}(\cdot | A) \).

1) \( \ell_1 \) iST: Algorithm 3 summarizes the \( \ell_1 \) iST channel estimation obtained by combining the conditional \( \ell_1 \) iST and AAD techniques. The EM sub-problems (15) and (16) are iteratively performed at Steps 5 and 7, respectively. The function \( f^{\ell_1}_{PCA}(\cdot | A) \) is the conditional \( \ell_1 \) iST technique shown in Algorithm 2, where it represents the \( \ell_2 \) iST channel estimation in the initial iteration since \( A^{[0]} = \{ 1, \ldots, W \} \). The input parameter \( r_{d_m} \geq 0 \) is used to prevent under-estimating the delay profile (43).

The optimal solution \( \hat{H}^{[n]} \) to the problem (12) is determined from all the possible \( N_{AAD} \) candidates, where the index \( n \) may be chosen according to the minimum of AICc:

\[
\text{AICc}(\hat{H}^{[n]}) = 2 L (\ell, \hat{H}^{[n]} + 2 K_{free}(K_{free}+1) / (N_{free} - K_{free} - 1). \)
\]

(47)

The parameters \( K_{free} \) and \( N_{free} \) are given by \( K_{free} = \sum_{k=1}^{K} R_{S,k} r_{T,k}^{[n]} \) and \( N_{free} = L_{a} \), respectively. The output \( \theta^{[n]}_k \) is reused as the input parameters \( \theta^*_k \) at the next slot timing. However, we do not update the original \( \theta^*_k \) in the for-loop between Steps 3 and 8.

**F. Computational Complexity Order**

Table II summarizes complexity orders needed for the proposed algorithms. The \( \ell_1 \) iST and \( \ell_1 \) pST algorithms increase the complexity order by \( O(k^3 N_{AAD}) \) and \( O(k^6 N_{AAD}) \), respectively, from the order \( O(k^6 + k^2) \) needed for the conventional \( \ell_2 \) pST technique, where \( k \def = \max \{ W, N_T, N_R \}. \)

\[\text{Algorithm 3 The} \ell_1 \text{iST channel estimation.} \]

**Input:** \( Y, \sigma^{2}_n, \Gamma, \theta_R, \theta_T, \) and \( r_{d_m} \).

1: Initialize \( A^{[0]} = \{ 1, \ldots, W \} \).
2: Compute \( R_{YY} \) and update \( K^{[n]}_{\ell} \).
3: for \( n = 0 \) to \( N_{AAD} \) do
4: Set \( r_{d_m} \) at 0 when \( n = N_{AAD} \), otherwise \( r_{d_m} = r_{d_m} \).
5: \( \{ \hat{H}^{[n]}, \theta^{[n]}_{\ell_1} \} \) \( \def = f^{\ell_1}_{PCA}(Y, R_{YY}, \theta_R, \theta_T, r_{d_m}, |A^{[n]}|). \)
6: Update \( \theta^{[n]}_k \) by using \( \hat{H}^{[n]} \).
7: \( \hat{A}^{[n+1]} = \text{AAD}(\hat{H}^{[n]}, \theta^{[n]}_{\ell_1}, \sigma^{2}_n, |A^{[n]}|) \) by (42).
8: end for
9: \( \hat{n} = \arg \min_{n \geq 1} \text{AICc}(\hat{H}^{[n]}) \) by using (47).

**Output:** \( \hat{H}^{[n]}, \theta^{[n]}_{\ell_1}, \) and \( \theta^{[n]}_k \).

1) \( \ell_1 \) iST : Table III details the complexity order required for the \( \ell_1 \) iST. The first seven items\(^8\) in Table III are the complexity for Algorithm 2 \( f^{\ell_1}_{PCA}(\cdot | A^{[n]}) \) performed at Step 5 in Algorithm 3, whereas the last item describes the complexity for the AAD algorithm performed at Step 7 of Algorithm 3.

As observed from Table III, the complexity for the \( \ell_1 \) iST is dominated by that needed to update the CCM\(^9\) \( K[\hat{G}^{[n]}_{\ell}] \) and the projection matrices \( \hat{\Phi}_i, \) where \( 0 \leq r_{d_m} \leq W. \)

2) \( \ell_2 \) iST: The complexity for the \( \ell_2 \) iST is also determined by Table III with two modifications: 1) the CCM \( K[\hat{G}^{[n]}_{\ell}] \) can be updated in \( O(N_T \cdot W^3) \) for the fixed active-set \( A^{[0]} = \{ 1, \ldots, W, N_T \} \) according to the definition of the operation \( K[\cdot] \). 2) Only the first AAD iteration \( (N_{AAD} = 1) \) is performed.

3) Unstructured-iST: As a benchmark, we summarize the complexity of the conventional approach [16] referred to as unstructured-iST (u-iST). It also performs the ST-subspace projection for the \( \ell_1 \) LS channel estimate as in (29). However, as discussed in [16], the projection matrix is approximated by that obtained by the \( \ell_2 \) iST. The complexity order needed for the u-iST technique is, hence, the same as that of the \( \ell_2 \) iST.

4) \( \ell_1 \) pST: Table IV details the complexity order for the \( \ell_1 \) pST. The first six items in Table IV are, similar to Table III, the complexity required for the function \( f^{\ell_1}_{PCA}(\cdot | A^{[n]}) \). As shown in Table IV, the complexity for the \( \ell_1 \) pST is dominated by that to update the spatial CCMs \( K[G^{[n]}_{\ell}] \) with the complexity order \( O(N_T \cdot W^2 N_R) \) and \( O(N_R \cdot W \cdot N_T^2) \), respectively, only for the initial active-set \( A^{[0]} = \{ 1, \ldots, W, N_T \} \).

**V. Performance Analysis**

MSE performance of the proposed algorithm is shown after detailing the estimate error \( \Delta \hat{H}_A \equiv \hat{H}_A - H. \) According to [30], the vectorized error \( \text{vec}(\Delta \hat{H}_A) \) of the estimate (40) can be decomposed, as

\[
(48)
\]

\[\text{vec}(\Delta \hat{H}_A) = (J_A \otimes I_{N_T}) (\Delta \alpha(\cdot) + \Delta \Gamma(\cdot)) - \text{vec}(H^\perp_{\ell}). \]

\(^8\)The complexity needed to obtain the \( \ell_1 \) LS estimates \( \hat{G}^{[n]}_{\ell} \) is dominated by \( O(W^3 N_T^2) \) in \( N_{AAD} \) iterations [17], [29].

\(^9\)The complexity needed to update the CCMs \( K[\hat{G}^{[n]}_{\ell}] \) is dominated by \( O(W^3 N_T^2) \) in \( N_{AAD} \) iterations [17], [29].
where $H_A$ is the CIR unsupported with the active-set $A$ and
\[
\Delta_\Omega(A) = \hat{\Pi}(A) \text{vec}\{\Delta \hat{G}_A^{LS}\}, \quad (49)
\]
\[
\Delta_{\Omega}(A) = \hat{\Pi}(A) - I_{|A||N_B|} \text{vec}\{G_A\}. \quad (50)
\]
The projection matrix $\hat{\Pi}(A)$ is $\bigoplus_{k=1}^{N_T} \hat{\Pi}_k(A)$, where
\[
\hat{\Pi}_k(A) = \left[ (Q_{A,k}^{-1} \mathcal{V}_k) \otimes I_{N_B} \right] \hat{\Phi}_k \left[ \mathcal{V}_k^T \otimes I_{N_B} \right].
\]

**Proposition 5** (Symbol-wise error variance bound).
\[
\text{diag}\left\{ \mathbb{E}[\Delta \hat{G}_A] \right\} \succeq \text{diag}\left\{ K \Delta \hat{G}_A (\sigma_{\Omega}^2, A_k, A) \right\}. \quad (51)
\]

*Proof.* According to (48),
\[
\mathbb{K} \left[ \Delta \hat{G}_A \right] \succeq \mathbb{K} \left[ \text{mat}_{N_B} \{ \text{vec}(X) \} \right], \quad (52)
\]
where $A \succeq B$ denotes that the residual $A - B$ is a positive semi-definite matrix and the operation mat$_N\{\cdot\}$ performs inversion of the vectorization: mat$_N\{\text{vec}(X)\} = X \in \mathbb{C}^{N \times M}$.

**TABLE III**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Eqn.</th>
<th>Complexity</th>
<th>Exec. Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[G_{A</td>
<td>k}^{LS}]$</td>
<td>(18)</td>
<td>$0(N_T^2 N_B^2)$</td>
</tr>
<tr>
<td>$V_{A,k}$</td>
<td>(23)</td>
<td>$0(N_T \cdot W^3)$</td>
<td>NAAD</td>
</tr>
<tr>
<td>$B_{T,k}$</td>
<td>(28)</td>
<td>$0(N_T \cdot N_B^2)$</td>
<td>NAAD</td>
</tr>
<tr>
<td>$G_{A,k}$</td>
<td>(29)</td>
<td>$0(N_T \cdot N_B^2)$</td>
<td>NAAD</td>
</tr>
<tr>
<td>$A^{[n+1]}$</td>
<td>(42)</td>
<td>$0(N_T \cdot W^3)$</td>
<td>NAAD</td>
</tr>
</tbody>
</table>

This is because $\Delta \hat{G}_A$ is dominated by
\[
\text{mat}_{N_B} \{ \Delta_0(A) \} = \Delta \hat{\Theta} \left[ \bigoplus_{k=1}^{N_T} \mathcal{V}_k \right] Q_{A,k,k}^{-1} H_{k,n}^{-1}, \quad (53)
\]
where we use (29) and $\Delta \hat{\Theta} = [\Delta \hat{\Theta}_1, \cdots, \Delta \hat{\Theta}_{N_B}]$ with
\[
\Delta \hat{\Theta}_k = \Gamma^{-1/2} \text{mat}_{N_B} \left\{ \hat{\Phi}_k \text{vec}\{\Delta \hat{G}_A^{LS} \hat{\mathcal{V}}_k \} \right\}.
\]
Moreover, we have
\[
\mathbb{K} \left[ \Delta \hat{\Theta} \right] \succeq \bigoplus_{k=1}^{N_T} \mathbb{K} \left[ \Delta \hat{\Theta}_k \right] = \bigoplus_{k=1}^{N_T} \sigma_{\Omega}^2 A_{k,k}. \quad (54)
\]

It is because $\mathbb{E}[\Delta \hat{\Theta}^{[1]}(\Delta \hat{\Theta})]$ is $\sigma_{\Omega}^2 A_{k,k}$ for $i = j$, otherwise 0, where $\Delta \hat{\Theta}_{k,i}$ is the $i$-th column vector of $\Delta \hat{\Theta}_k$. Therefore, (51) is obtained from (52), (53), and (54). \qed

**Proposition 6** (MSE performance of the $\ell_1$ ST algorithm).
\[
\text{MSE}(\hat{H}^{[1]}_{ST}) = \min_A \text{MSE}(\hat{H}_A | A) \geq \sum_{k=1}^{N_T} \mu(\sigma_{\Omega}^2, r_{S,k}, r_{T,k}, A_k^2) + \beta(r_{S,k}, r_{T,k}) \quad (55)
\]
with $\beta(r_{S,k}, r_{T,k}) = \mathbb{E}[||\mathbf{H}_k||_2^2] - \sum_{j=1}^{r_{S,k}} \sum_{i=1}^{r_{T,k}} \lambda_j^2(\mathbb{E}[R_{b,i,j}])$, where the optimal active-set $A^*$ is determined by
\[
A^* = \arg \min_A \text{E}[||\Delta \hat{H}_A||^2] = \{ j | d_j \geq m_j(\sigma_{\Omega}^2, A), j \in A \}. \quad (56)
\]
The parameter $d_j$ is the $j$-th entry of the vector diag$\{\mathbb{K}[H]\}$.

*Proof.* The MSE (55) is obtained from (48), (51) and (46), where the bias errors $\mathbb{E}[||\Delta \hat{H}_A||_2^2] + \mathbb{E}[||\hat{H}_A||_2^2]$ is lower-bounded by $\beta(r_{S,k}, r_{T,k})$. Then we consider (56). By (48),
\[
\mathbb{E}[||\Delta \hat{H}_A||_2^2] \geq \mathbb{E}[||\Delta \hat{G}_A||_2^2] + \mathbb{E}[||\mathbf{H}_k||^2] = \mathbb{E}[||\mathbf{H}||^2] + 1^T_{W_{N_B}} \text{m}(\sigma_{\Omega}^2, A) - d_{H_A}, \quad (57)
\]
where $d_{H_A} = \text{diag}\{\mathbb{K}[H]\}$. Hence, we have
\[
A^* = \arg \max \sum_{j \in A} (d_j - m_j(\sigma_{\Omega}^2, A)), \quad (58)
\]
which is equivalent to (56). \qed

The AAD (42) is derived from the optimization (56) by substituting the estimate $d_{\mathbf{H}} = d_{\mathbf{H}} + \epsilon(\sigma_{\Omega}^2)$ for $d_{\mathbf{H}}$ and considering the error $\epsilon(\sigma_{\Omega}^2)$ in the LHS of the inequality.

**VI. NUMERICAL EXAMPLES**

**A. Simulation Setup**
We assume $N_T \times N_R = 3 \times 24$ and $6 \times 12$ MIMO transmission scenarios in unknown interference channels. The interference is caused by unknown transmitters in the neighboring service areas, where they also communicate with their base station (BS) using the same MIMO system setup as that of the target user. CIRs are generated according to the SCM [22], where six path fading channel realizations based on the Vehicular-A (VA) model with a 30 km/h mobility and the PB model with a 3 km/h mobility are used in the urban micro cell scenario. The path positions of the VA and PB models are respectively set at $\{1.0, 3.2, 6.0, 8.6, 13.1, 18.6\} + \Delta_\text{synch}$ and $\{1.0, 2.4, 6.6, 9.4, 17.1, 26.9\} + \Delta_\text{synch}$ symbol timings assuming that the TX bandwidth is 7 MHz with a carrier frequency of 5 GHz. The timing offset $\Delta_\text{synch}$ is set at 0 for the target user, whereas it is chosen randomly from the range

**TABLE IV**

<table>
<thead>
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<th>Exec. Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{a</td>
<td>k}^{LS}$</td>
<td>(18)</td>
<td>$0(W^3 N^2)$</td>
</tr>
<tr>
<td>$\mathbb{E}[G_{a</td>
<td>k}^{LS}]$</td>
<td>(30)</td>
<td>$0(N_T^2 W^3 + W^2 N^2)$</td>
</tr>
<tr>
<td>$V_{a,k}$</td>
<td>(23)</td>
<td>$0(N_T \cdot W^3)$</td>
<td>NAAD</td>
</tr>
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</tr>
</tbody>
</table>
[0.0, 3.0] for the unknown users. The maximum CIR length $W$ is, hence, set at 31. The TTI $\Delta_T$ is set at 2 slots.

Note that, as discussed in [16], the CIR matrix $H_k(l)$ following (2) can be obtained by re-sampling row-vectors of the complex channel gain $B_k(l)$. Concretely, the matrix $B_k(l)$ is generated with the SCM implementation [31] and the raised cosine filter of roll-off 0.3 is used to take account of the pulse shaping and the propagation delay of the multipaths models. Moreover, the antenna element spacing at the BS and the mobile station (MS) are set at 0.5 wavelength. The angle $\theta_{MS}$ between the BS-MS and the MS broadside [22] is fixed at $125^\circ$ for the target user. However, the angle $\theta_{ST}$ is chosen randomly from $[-180^\circ, 90^\circ]$ for the interference sources.

B. Channel Estimation Performance as SNR varies

The algorithms are verified with normalized MSE (NMSE) performance, where $\text{NMSE}(H) \triangleq \mathbb{E}[\|H-H\|^2]/\mathbb{E}[\|H\|^2]$ for estimates $H$. In this subsection, $S$ is set at a certain value, where we define the SIR and SNR by $\mathbb{E}[\|H\|^2]/\mathbb{E}[\|Z\|^2]$ and $\mathbb{E}[\|HX\|^2]/\mathbb{E}[\|X\|^2]$, respectively. The target user transmits signals over an unknown interference user, where the PB and VA models are assumed for the target and interference users, respectively. We refer to this TX scenario as PB/VA. The parameters in the AAD are set at $(N_{AAD}, E) = (3, [0.8W^2])$. The input parameter $r_{min}$ of Algorithm 3 is set at 1.

1) $ST$ vs. Temporal-Only: Fig. 3 shows NMSE performance of the $\ell_2 pST$ in the $6 \times 12$ MIMO system, where a TS of $L_t$ = 255 symbols is generated with the Gold sequence. The sliding window length $L_s$ in (17) is set at 50. A benchmark referred to as normalized adaptive-CRB (NaCRB) is also presented, where it is defined by normalizing (55) with $\mathbb{E}[\|H\|^2]$.

As depicted in Fig. 3, the $\ell_2 pST$ obtains a significant NMSE gain over an approach using the temporal subspace only ($\ell_2 T$-only) [26] at $\text{SIR} = 4 \text{dB}$. However, the NMSE gain decreases as SNR increases when there is no unknown interference. This observation confirms that, according to the property (10), the joint ST subspace-based estimators obtain the rank gain in a low to moderate SINR regime.

2) $iST$ vs. $pST$: Fig. 3 shows NMSE of the $\ell_2 iST$ technique, where the constant $r_{max}$ in (25) is set at $[0.5W]$. The $\ell_2 iST$ improves NMSE performance significantly over that of the $\ell_2 pST$ at the moderate $\text{SIR} = 4 \text{dB}$. This is because the $\ell_2 pST$ technique has difficulty to separate unknown interferes from the signal of interest, whereas the $\ell_2 iST$ improves the problem based on Proposition 1. However, the NMSE gain decreases in the negative SNR regime. This is because the CCM (28) of the ICA is computed from $1/W$ times fewer samples than that of the PCA.

3) NMSE details of the iST methods: We verify the NMSE performance of the iST algorithms according to (48). As shown in Fig. 4(a), the $\ell_2 iST$ estimates the average-rank $r_{iST}$ accurate enough compared with the true adaptive-rank based on Definition 1, where $r_{iST} \triangleq \sum_{k=1}^{N_r} \sum_{i=1}^{r_{iST}} r_{i,k}/N_r$. For a constant SIR, the interference-to-noise ratio (INR) is proportional to the SNR, which incurs an approximation error of (14). Hence, we observe from Fig. 4(a) that the rank estimates and the residual (49) are degraded in a high SNR regime.

However, in a low SNR regime, the $\ell_2 iST$ suffers from the projection error (50) significantly. This is because, as shown in (50), the projection error can be increased for a redundant active-set $A$. The $\ell_1 iST$ improves the projection error by iteratively pruning the active-set. As observed from Fig. 4(b), the AAD (42) gets the NMSE performance converged in three iterations. Of course, the $\ell_1$ regularization does not completely suppress unknown interference in the CCM. As shown in Fig. 3, the $\ell_1 iST$ asymptotically achieve the NaCRB if the subspace projection is estimated ideally given $H$.

C. Channel Estimation Performance as SIR varies

In this subsection, channel estimation algorithms are performed in the $3 \times 24$ MIMO system using the TS of $L_t$ = 127 symbols. $L_s$ = 100 is assumed. The CIRs follow the VA and PB models for the target user and two unknown interference users, receptively. We refer to the propagation scenario as VA/({PB, PB}) hereafter.

1) $\ell_1 pST$ vs. $\ell_1 iST$: The $\ell_1 pST$ is expected to improve the performance under unknown interference by leveraging the $\ell_1$ regularization as well as the $\ell_1 iST$. However, as shown in Fig. 5, the $\ell_1 pST$ does not outperform the $\ell_2 iST$ algorithm.

---

Footnotes:
10The interference terminals are not always synchronized to the receiver.
11We do not compute the response matrix $E_{i,k}$ in (2) to generate CIRs.
even in a low SINR regime. This is because Proposition 1 holds in the SCM-based channel realizations.

2) \( u \)-iST vs. \( \ell_1 \) iST: In a high SINR regime of \( SNR = 15 \) dB, the conventional \( u \)-iST [16] obtains equivalent NMSE performance\(^ {12} \) to that of the \( \ell_1 \) iST, since it is also the \( \ell_1 \) ST subspace-based channel estimation (12). However, at \( SNR = 0 \) dB, the \( u \)-iST technique is inferior to the \( \ell_1 \) iST in the entire SINR regime. This is because, in a low to moderate SINR regime, the \( u \)-iST inherits the projection accuracy problem, since it approximates the subspace projection with that obtained by the \( \ell_2 \) iST.

D. Comparison with Conventional Techniques

1) \( \ell_1 \) LS techniques: Fig. 6 shows NMSE performance of the conventional adaptive structured subspace pursuit (ASSP) technique [7] in the 6×12 MIMO system. The PB/VA scenario is assumed. INR is set at 0 dB. The TTI \( \Delta T \) is set at 1 slot in this subsection. As shown in Fig. 6, the ASSP outperforms the OMP since it improves accuracy of the active-set detection by using the maximum correlation in the past \( L_{ASSP} \) slots, where the optimal \( L_{ASSP} \) is chosen for 10. The stopping condition of the ASSP iteration is modified to inspect for each TX stream so that it approximately achieves the \( \ell_1 \) LS NMSE bound.\(^ {13} \) However, the ASSP does not achieve the NaCRB since it does not consider the spatial subspace.

2) \( \ell_1 \) LS + iST-subspace method: The \( \ell_1 \) iST may be composed of an arbitrary \( \ell_1 \) LS and the conditional \( \ell_1 \) MMSE channel estimation techniques. However, such a naive extension does not always achieve the optimal performance. Fig. 6 shows the NMSE performance of an iST subspace-based compressive channel estimation constructed with the ASSP and (15). We refer to it as ASSP + iST, hereafter. The ASSP + iST does not outperform the \( \ell_2 \) iST, although the ASSP follows the \( \ell_1 \) LS MMSE bound in the MIMO channels.

Fig. 7 details the NMSE performance of the ASSP + iST, where SNR is set at 6 dB. It is observed from Fig. 7 that the NMSE performance of the ASSP + iST diverges from that of the \( \ell_1 \) iST as SINR decreases. We find that, in both algorithms, the iST-subspace analysis is performed as expected since the average ranks \( \hat{r}_{iST} \) are estimated relevantly. The residual terms \( \mathbb{E}[\| \Delta L_i \|^2] \) (49) and the projection errors \( \mathbb{E}[\| \Delta H_i \|^2] \) (50) are also computed equivalently in the two algorithms. We can find from Fig. 7 that, however, the ASSP + iST increases the bias error \( \mathbb{E}[\| \Delta H_i \|^2] \) in the low SINR regime. This is because the ASSP + iST selects the active-set under the \( \ell_1 \) LS criterion, which can deteriorates accuracy as the \( \ell_1 \) LS MMSE estimates when channels are not exactly sparse.

3) Frequency domain CS-based estimation: The simultaneous weighted OMP (SW-OMP) algorithm [11] can be performed by transforming the received TS matrix after the spatial whitening into the frequency domain:

\[
\Gamma^{\frac{1}{2}} Y(l) \cdot F^T = \Gamma^{\frac{1}{2}} H_F(l) \Lambda_p(l) + \Gamma^{\frac{1}{2}} \eta(l) F^T,
\]

where \( F = \frac{1}{\sqrt{L_i}} [(\cos(\pi i/j - 1) \sqrt{-1})]_{i,j} \) is the \( L_i \times L_i \) discrete Fourier transform (DFT) matrix. We denote \( [(a(i,j))]_{i,j} \) as a matrix whose \( (i,j) \)-th entry is \( a(i,j) \). The matrix \( H_F(l) \) is \( [H_1(l), \ldots, H_{N_T}(l)](1_{N_T} \otimes F^T) \) with \( H_k(l) = [H_k(l,0), \ldots, H_k(l,-N_T)] \) for \( k \in \{1 : N_T\} \). The \( N_T L_i \times L_i \) block diagonal matrix \( \Lambda_p(l,j) \) is defined by (58) assuming known active-sets and \( \Pi = I \).

For the \( f \)-th bin, the \( H_f(l) \) is given by (58), where \( H_f(l,j) = A_R R_f(l,j) \Lambda_f(l) (\forall j \in \{1 : L_M\}) \) is used, where \( R_f(l,j) = \mathbb{E}[\| \mathbf{H}_f(l,j) \|^2] \) in the quantized angular domain representation. The quantized receive antenna response matrix \( A_R \) is given by \( \mathbb{E}[\| \mathbf{H}_f(l,j) \|^2] \) for the \( f \)-th bin. Let the coherent time be longer than \( L_M \) slots.

\[
H_f(j) \approx A_R H_f(l,j) \Lambda_f(l)(\forall j \in \{1 : L_M\}) \quad (58)
\]

is obtained, where \( A_R \) is determined by the SW-OMP. However, the SW-OMP exhibits a MSE floor in a high SINR regime of \( SNR = 15 \) dB, the conventional \( u \)-iST technique is inferior to the \( \ell_1 \) iST in the scenario. Moreover, the SW-OMP exhibits an MSE floor in a high SINR regime since the approximation (58) is inaccurate in this scenario. Therefore, the SW-OMP is inferior to the \( \ell_2 \) iST in a moderate SINR regime, although the oracle stopping criterion is assumed. According to our experiment, increasing the resolution \( (G_R, G_T) \) improves the problem insignificantly.

\[^{12}\text{Technically, the } u \text{-iST can achieve slightly better NMSE than the } \ell_1 \text{iST in a high SINR regime, since the bias error of the } u \text{-iST can be less than that of the } \ell_1 \text{iST [16]: } \mathbb{E}[\| \hat{\mathbf{H}}(l) - \mathbf{H}(l) \|^2] \leq \mathbb{E}[\| \mathbf{H}(l) - \mathbf{H}(l) \|^2] \text{ with } \mathbf{H}(l) = \{1 : W_{N_T}\} \text{ holds when the subspace projection is accurate enough.}

\[^{13}\text{It can be defined by (57) assuming known active-sets and } \Pi = I.}
This is because, unlike the iST methods, cancel co-streams in (24), the SW-OMP does not solve the MAI problem directly.

E. BER Performance

1) BER in Low SINR: Fig. 8 shows BER performance in the 3 × 24 MIMO system. An $L_b = 2048$ bit binary data sequence is turbo encoded with rate $R_c = 1/3$ by using the transfer function $[1, g_1/g_0]$ with $(g_0, g_1) = (13, 15)$. The channel-encoded sequence is mapped to $N_T R_c L_b / N_c$ orthogonal frequency division multiplexing (OFDM) symbols with $N_c = 1024$ subcarriers. We transmit an OFDM-symbol followed by the TS section. The VA/PB/PB scenario is assumed at SNR = 0 dB.

As shown in Fig. 8, the receiver achieves BER = $10^{-5}$ at SIR = -10 dB if CIR matrix $H$ is known perfectly. We can find from Fig. 5 that, however, the NMSE of channel estimates is greater than $10^{-1}$ in the negative SIR regime when SNR is set at 0 dB. Hence, the receiver using actual channel estimates has a large BER performance gap from the ideal receiver. For example, the receiver using the ASSP estimation does not achieve BER = $10^{-5}$ in the negative SIR region. The receiver in a large-scale MIMO system is, hence, necessary to jointly utilize the CS and ST subspace-based approaches. As shown in Fig. 8, the receiver using the $\ell_1$ pST improves BER more than 3 dB over that of the ASSP.

According to Proposition 1, the iST methods are expected to improve receiver performance. However, it is observed from Fig. 8 that the receiver with the $\ell_2$ iST suffers from a BER floor due to the subspace projection error. The projection accuracy can be improved by using $\ell_1$ regularization since the CIRs are observed as asymptotically sparse parameters in the low SIR regime. As observed from Fig. 8, the receiver with the ASSP+iST technique solves the BER floor by using the proposed $\ell_1$ iST algorithm, the receiver further improves the BER performance by 1 dB over that of the ASSP+iST. This is because the AAD algorithm optimally selects the active-set by taking the ST-subspace into account.

2) BER with Interference Variation: We verify the proposed algorithms in a practical situation where the unknown interference changes abruptly. The variation of interference is set at $\Delta SIR \pm 5$ dB in every 100 slot-interval, where the $\Delta SIR$ denotes the average of an SIR configuration. SNR is fixed at 6 dB. The PB/VA scenario is used in 6 × 12 MIMO channels.

As shown in Fig. 9, the receiver using the $\ell_2$ iST exhibits a BER floor even in a moderate SINR regime. This is because the $\ell_2$ iST suffers from the projection error due to the abrupt SIR variation. As above-mentioned, CS-based techniques can improve the projection accuracy problem. However, as observed from Fig. 9, the ASSP+iST technique does not outperform the $\ell_2$ iST. This is because the CIRs generated from the PB model are not observed as exactly sparse channels in a moderate to high SINR regime.

The AAD algorithm can improve BER performance even in the approximately sparse PB channels, since it is designed to adaptively minimize the MSE performance according to the sparsity of the interested signals. The conventional u-iST performs the EM algorithm $\{15, 16\}$ by using the $\ell_2$ iST and the AAD, respectively. Nevertheless, the u-iST technique does not completely solve the BER floor problem since it inherits the projection accuracy problem from the $\ell_2$ iST.

Hence, it is necessary to exactly perform the EM algorithm. As shown in Fig. 9, the receiver with the $\ell_1$ pST improves receiver performance at BER = $10^{-5}$ over that of the $\ell_2$ iST, since it performs the E-step (15) correctly. According to Proposition 1, the receiver using the $\ell_1$ iST algorithm further...
improves BER than that of the $\ell_1$ PSST, and it achieves a BER gain of 2.5 dB in SIR over that of the $\ell_2$ PSST.

VII. CONCLUSIONS

The ordinary $\ell_2$ PSST channel estimation technique based on the ICA improves MSE performance over the $\ell_2$ PSST using the PCA. However, a receiver using the $\ell_2$ PSST can suffer from a BER floor problem in unknown interference MIMO channels. As a solution to the problem, we proposed a new $\ell_1$ PSST algorithm which suppresses unknown interference in the CCM by leveraging the temporally sparse property of the observed CIRs. Hence, the $\ell_1$ PSST accurately performs a spatial-ICA for independent path components.

Simulation results shown in this paper verify that the proposed algorithm achieves a significant MSE gain by using the AAD algorithm that detects the active-sets optimally. Hence, a receiver using the $\ell_1$ PSST channel estimation solves the BER floor problem in unknown interference MIMO channels.

ACKNOWLEDGEMENTS

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